Homotopically trivializing the circle in the framed little disks

Gabriel C. Drummond-Cole

Northwestern University

March 13, 2012

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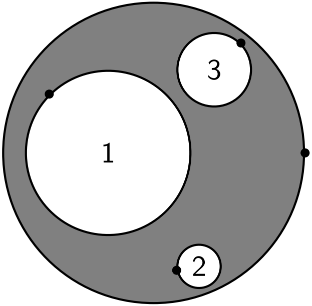

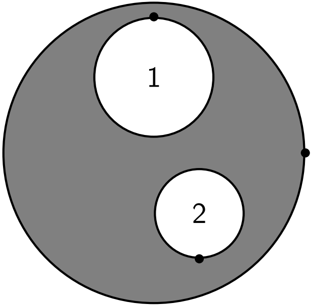

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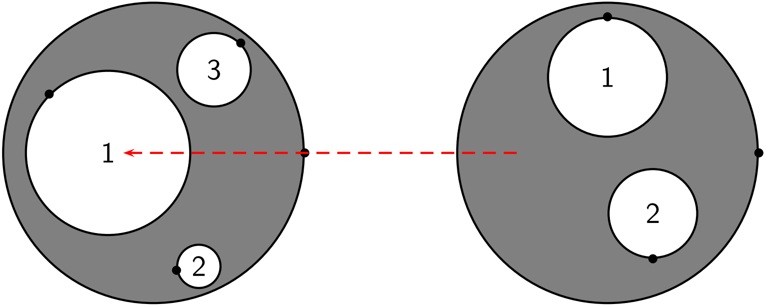

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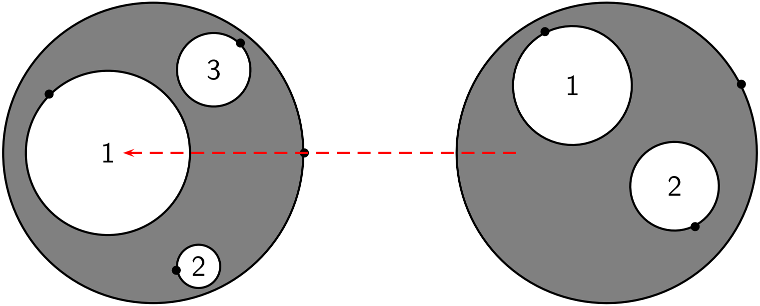

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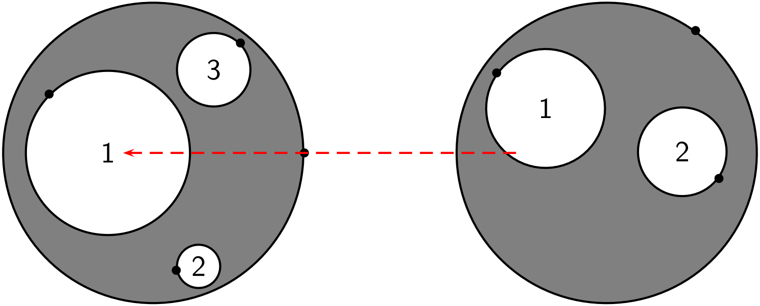

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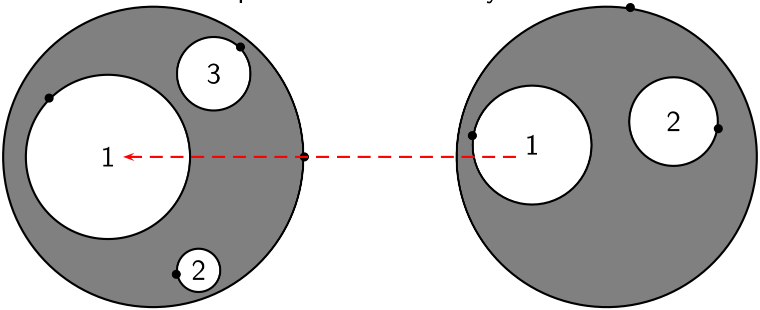

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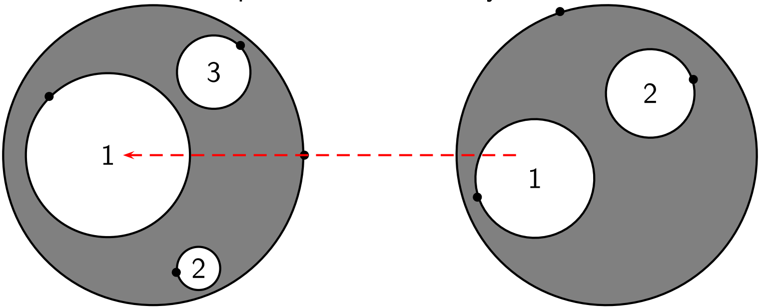

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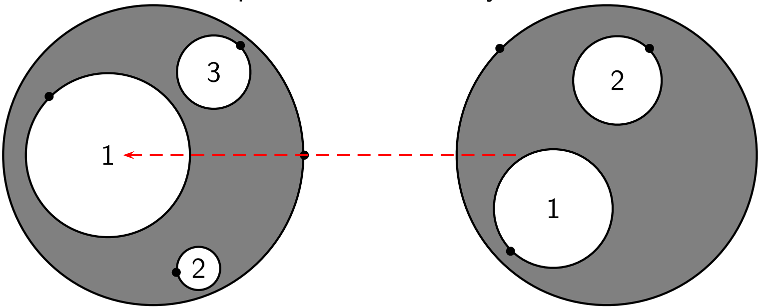

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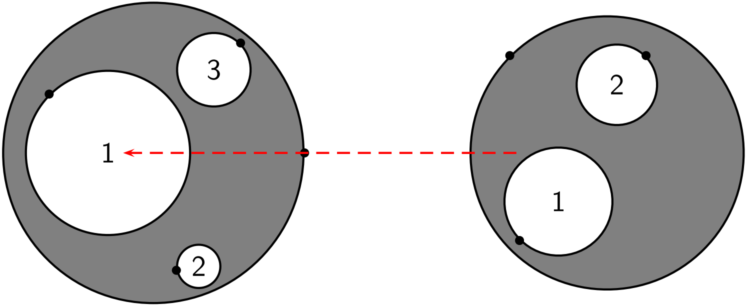

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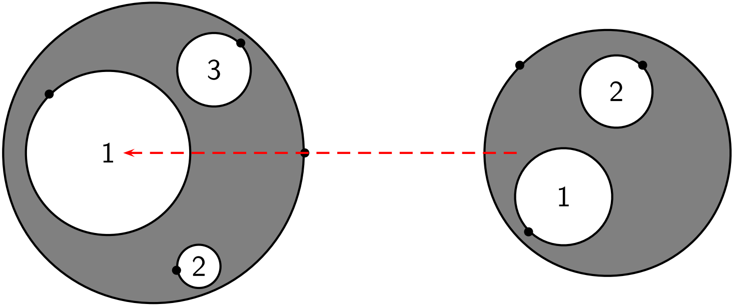

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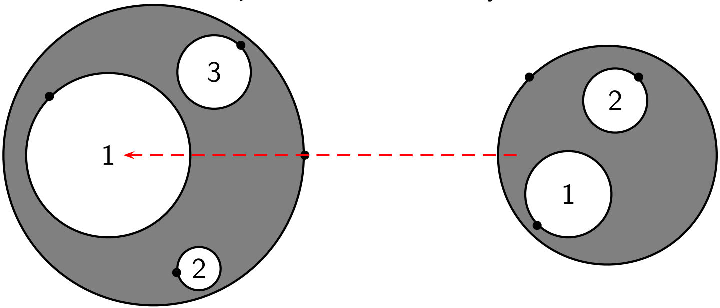

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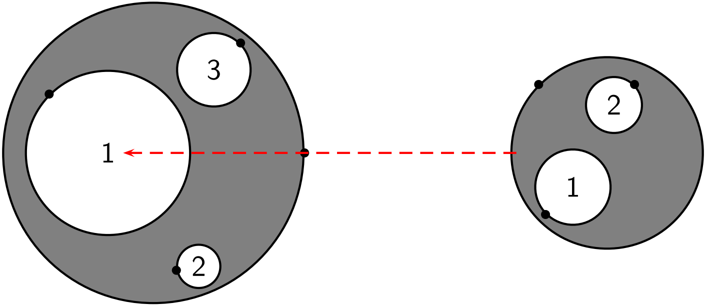

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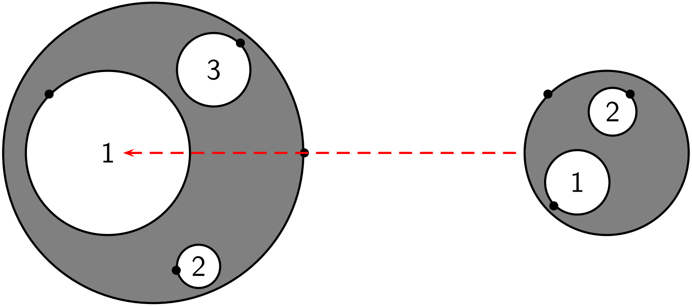

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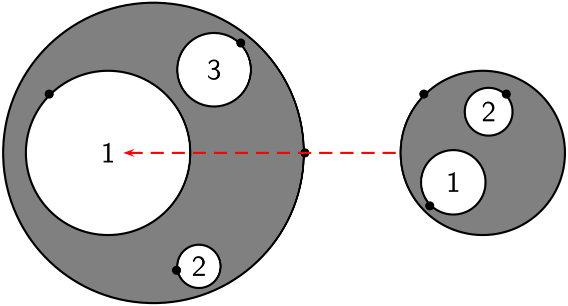

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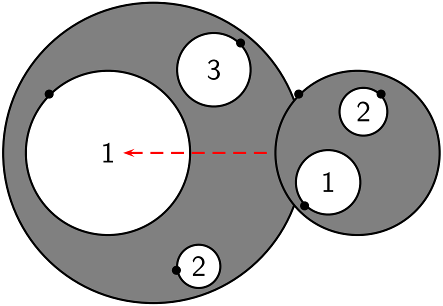

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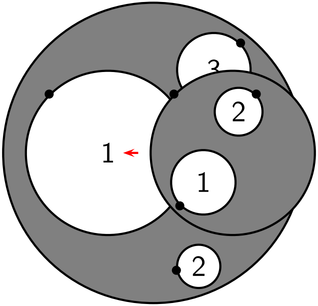

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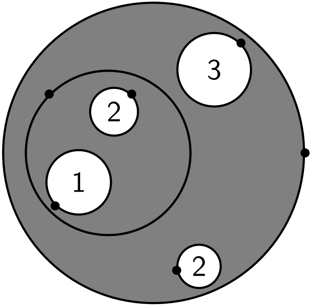

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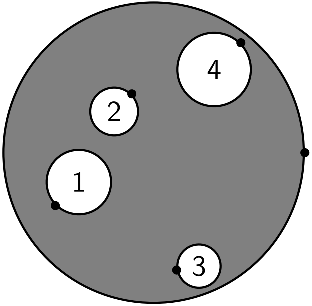

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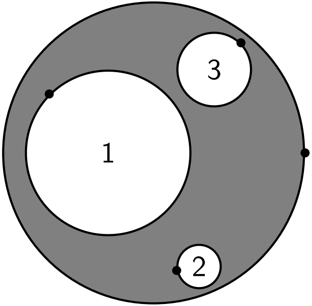

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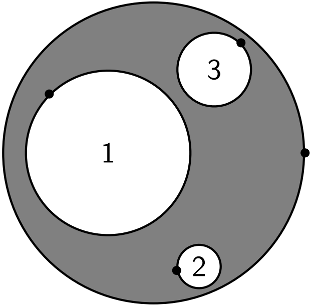

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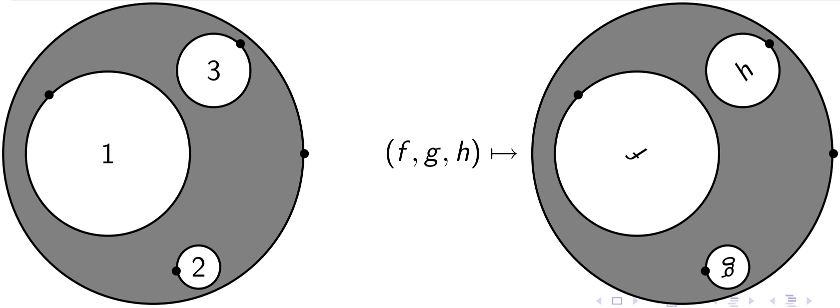

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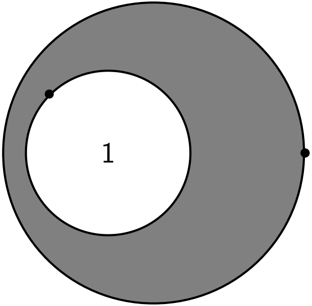

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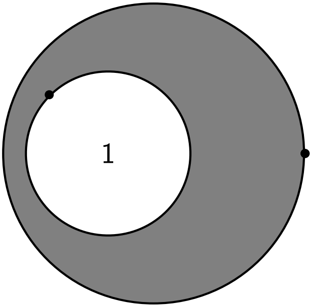

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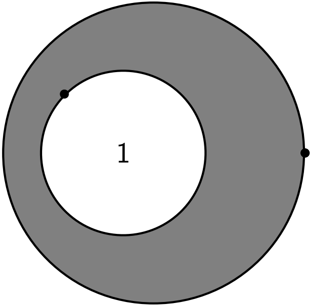

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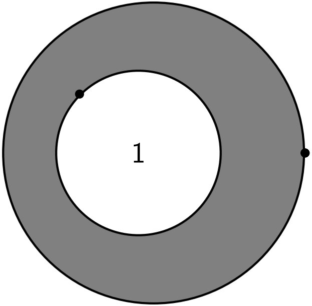

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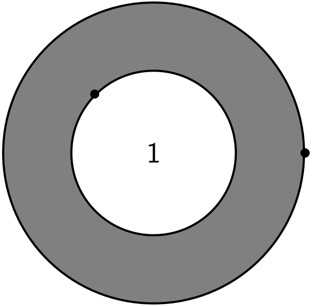

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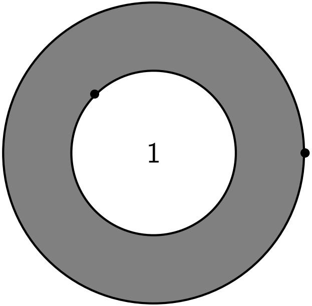

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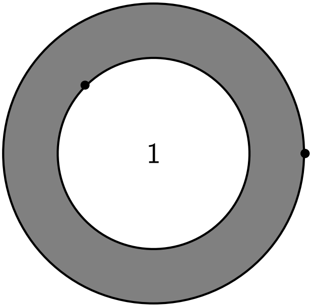

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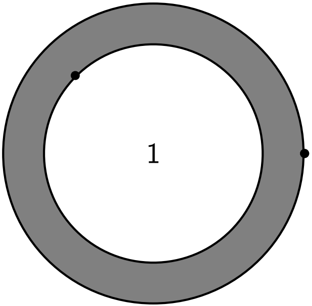

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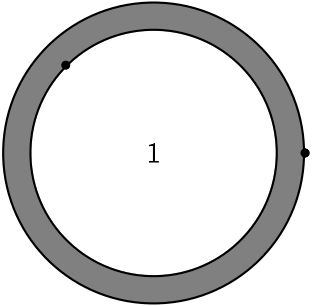

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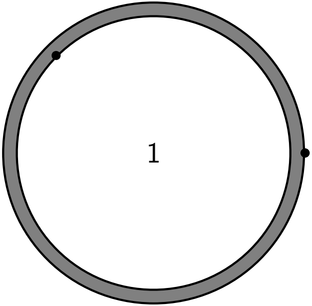

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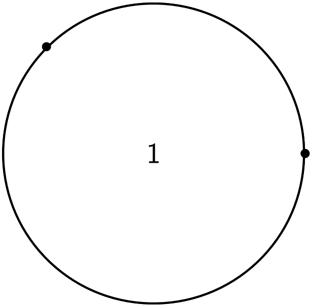

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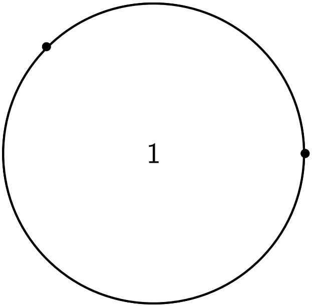

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Trivializing the circle

What happens if a space is acted on by the framed little disks but the circle acts trivially? In such a case, how does this change the space of operations?

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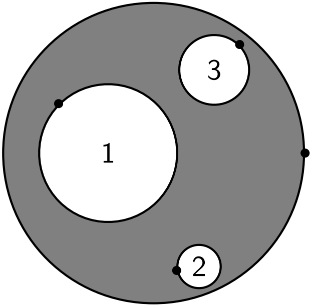

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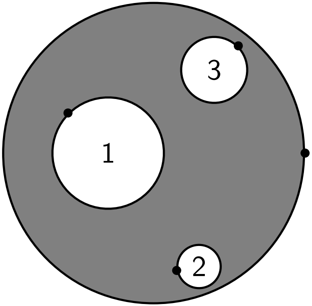

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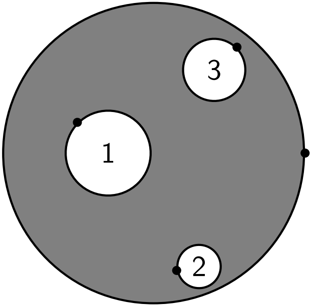

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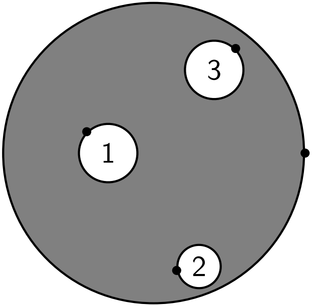

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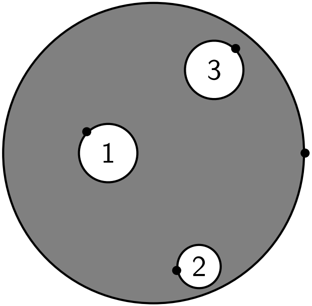

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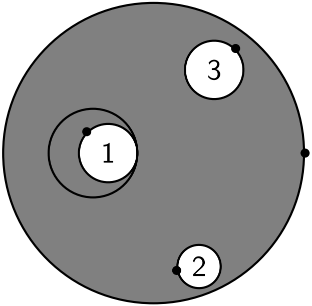

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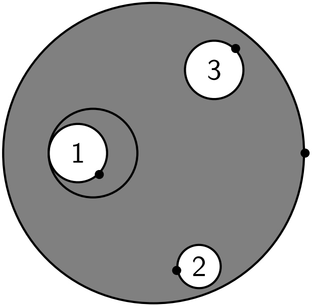

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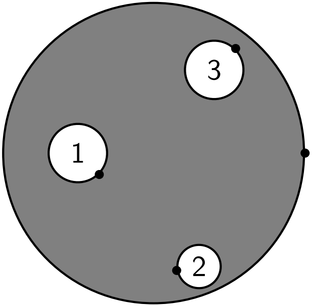

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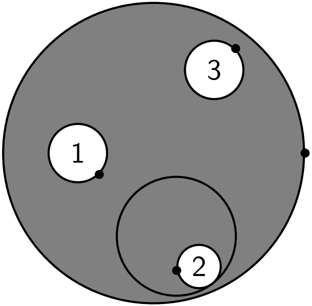

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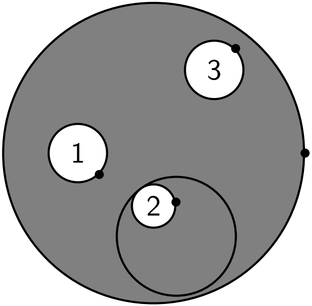

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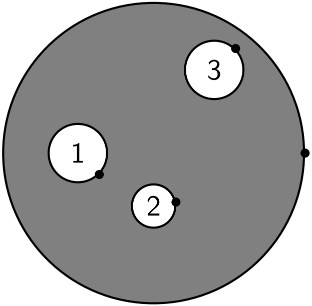

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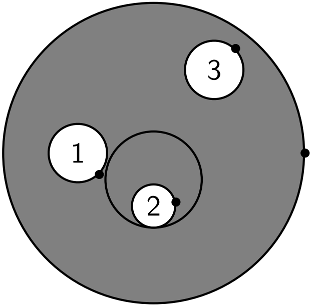

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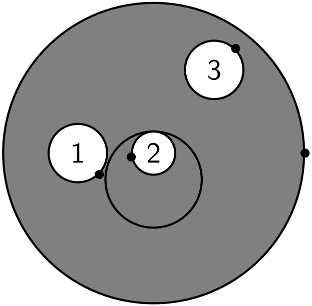

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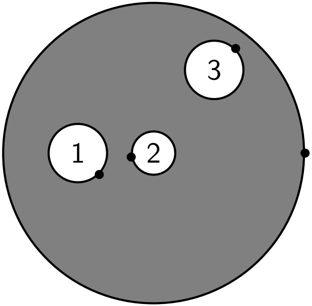

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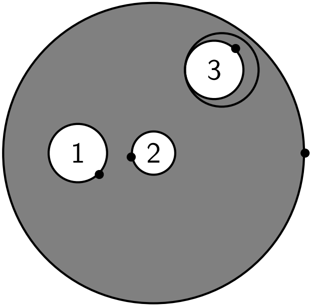

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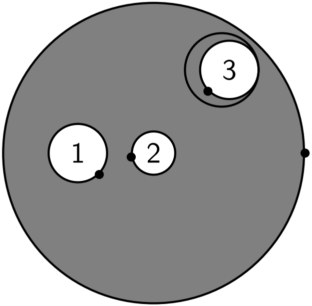

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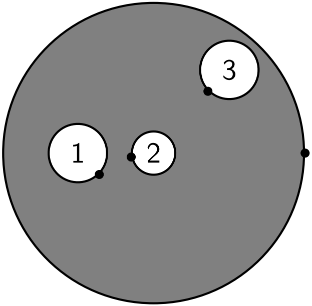

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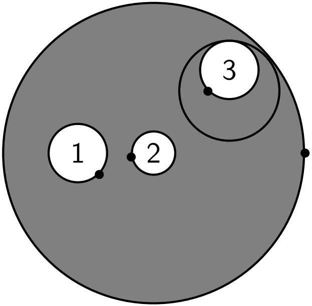

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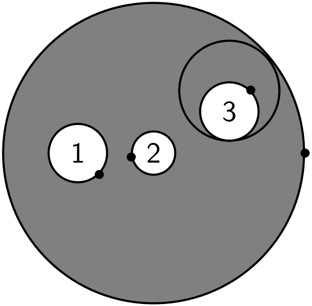

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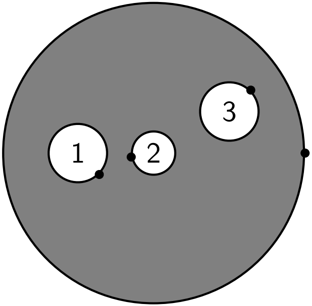

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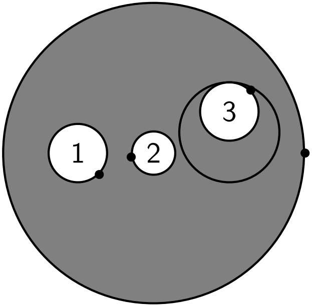

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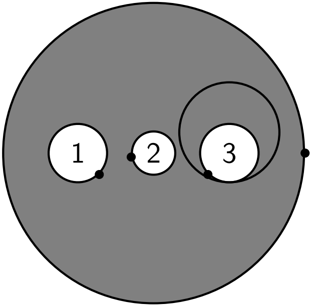

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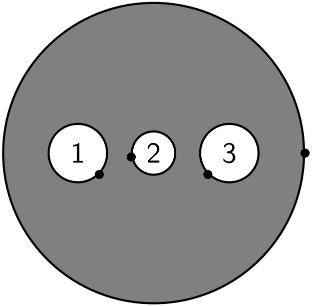

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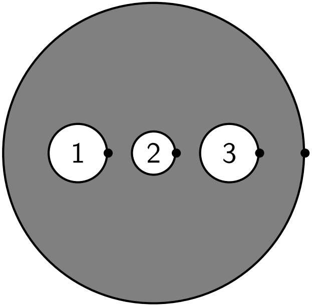

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Example

Space of operations: the disk D^2, with the standard multiplication in $\mathbb{C}$ and the standard circle action.

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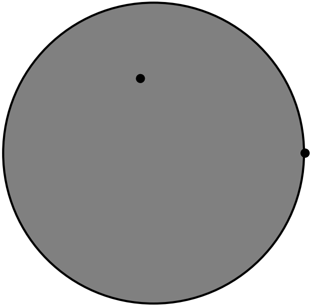

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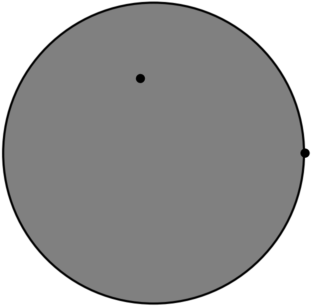

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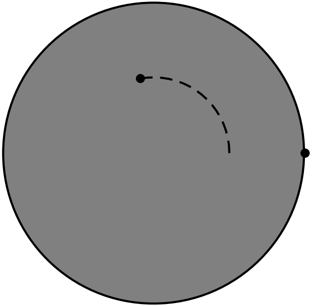

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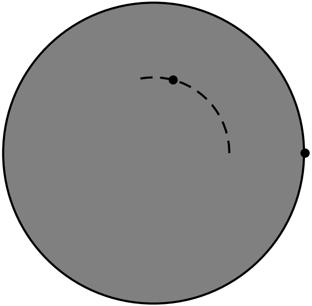

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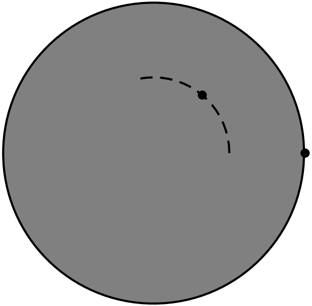

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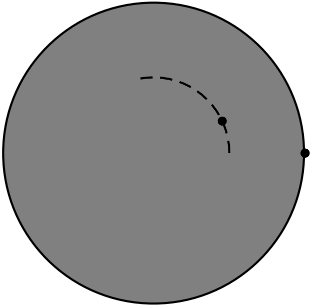

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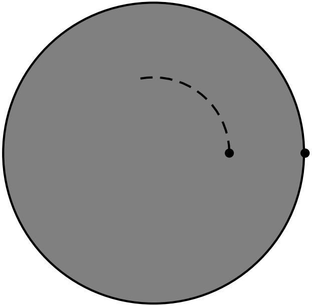

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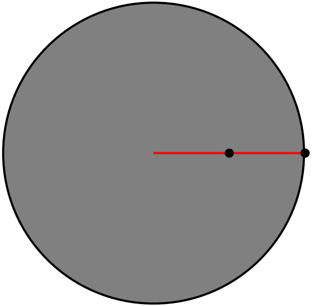

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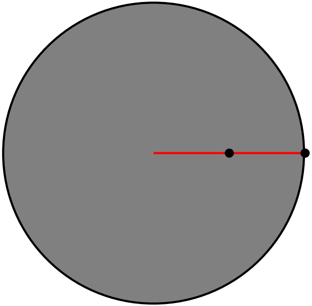

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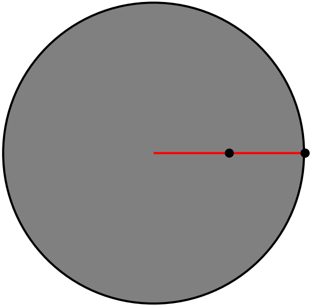

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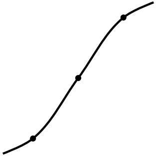

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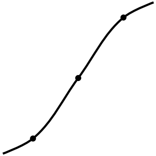

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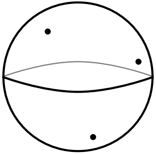

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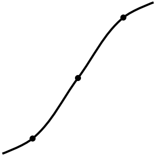

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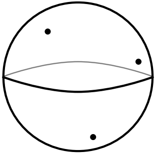

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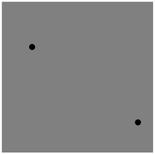

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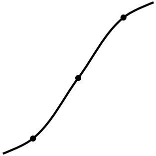

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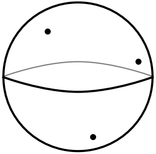

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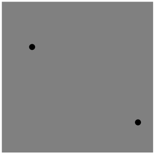

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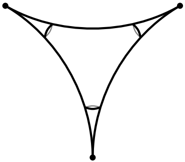

Hyperbolic

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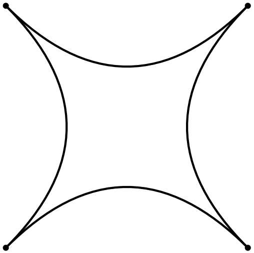

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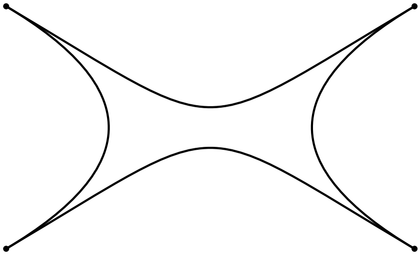

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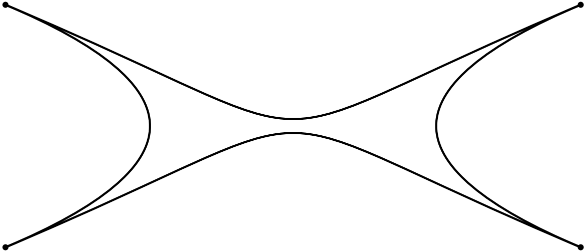

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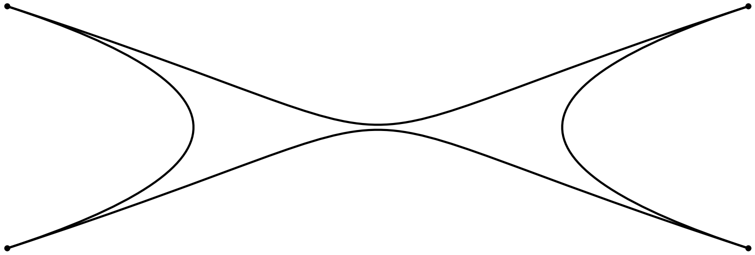

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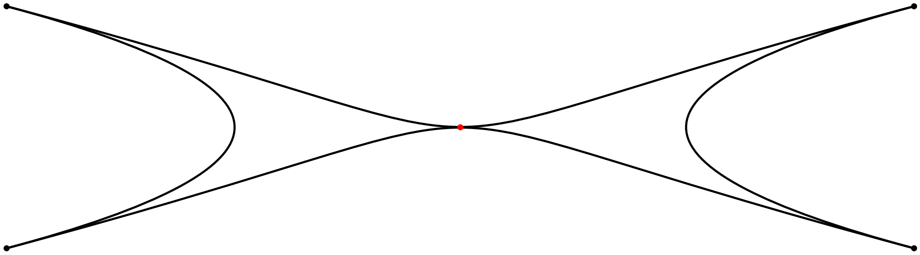

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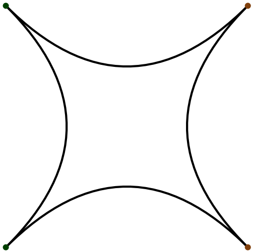

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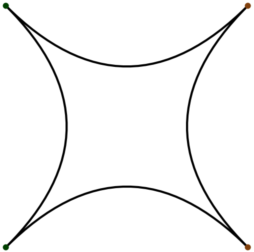

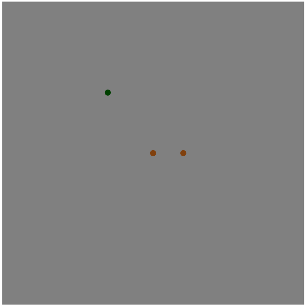

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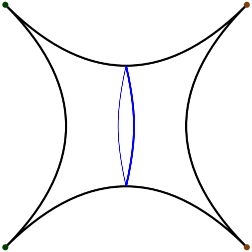

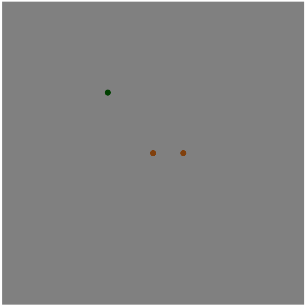

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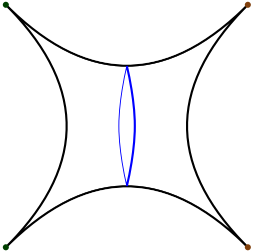

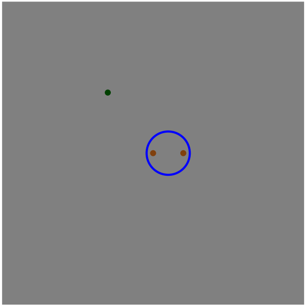

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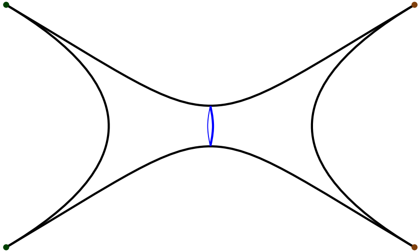

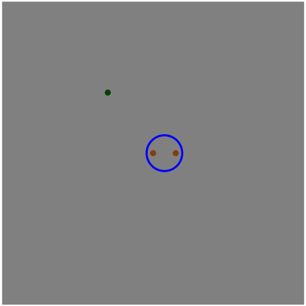

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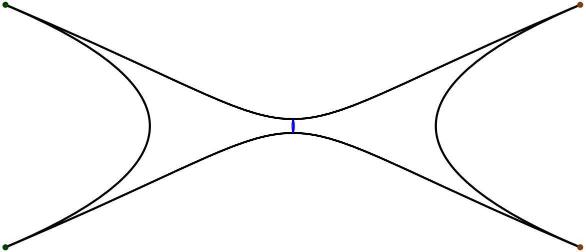

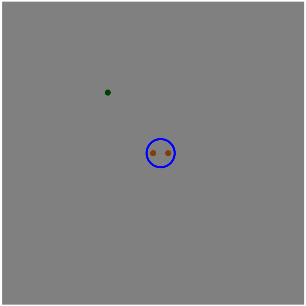

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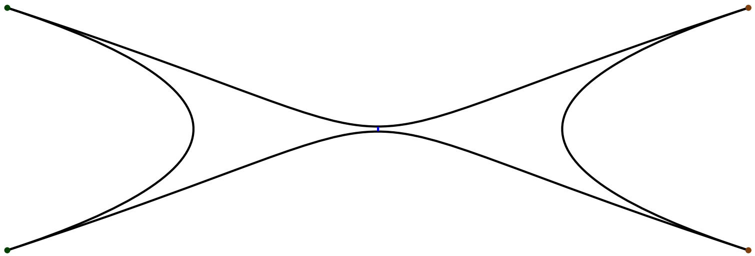

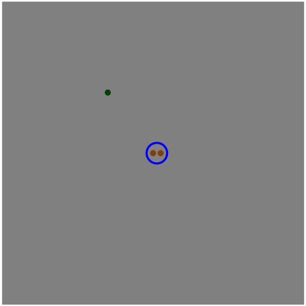

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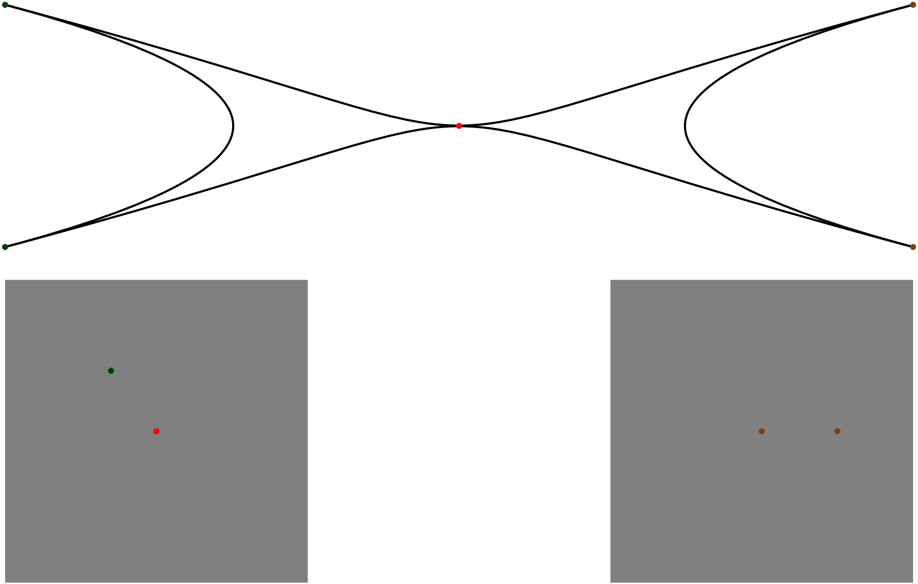

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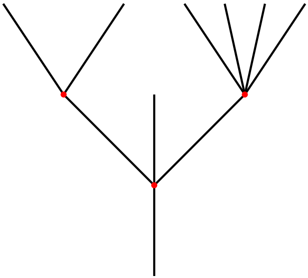

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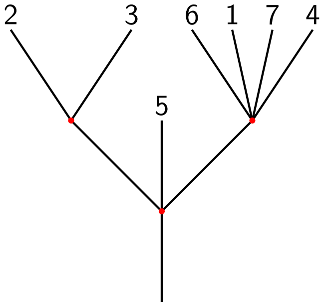

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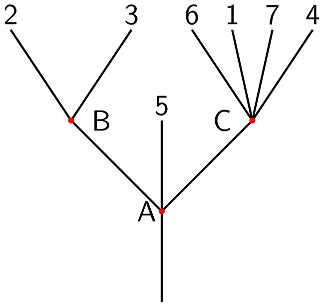

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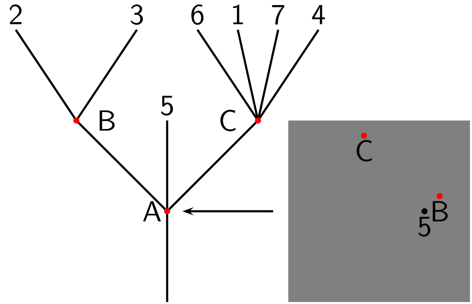

Sketch of Definition

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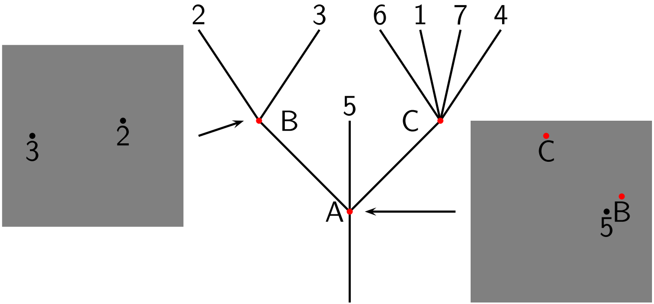

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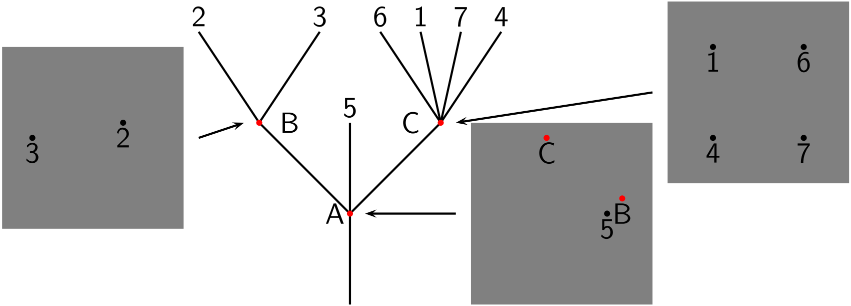
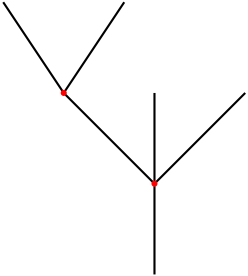


Composition in $\overline{\mathcal{M}}_{0,n}$



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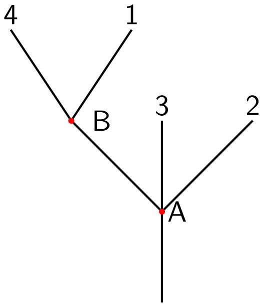

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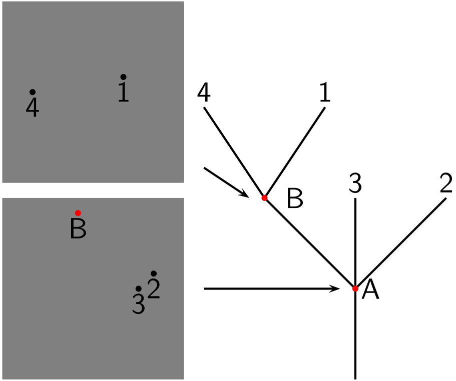

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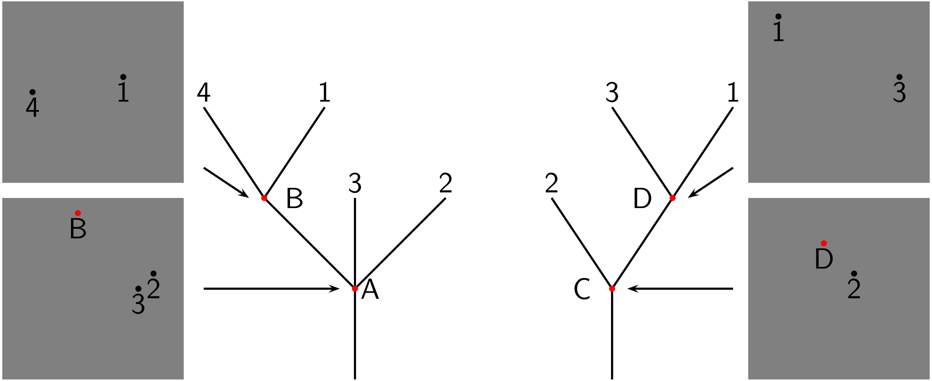

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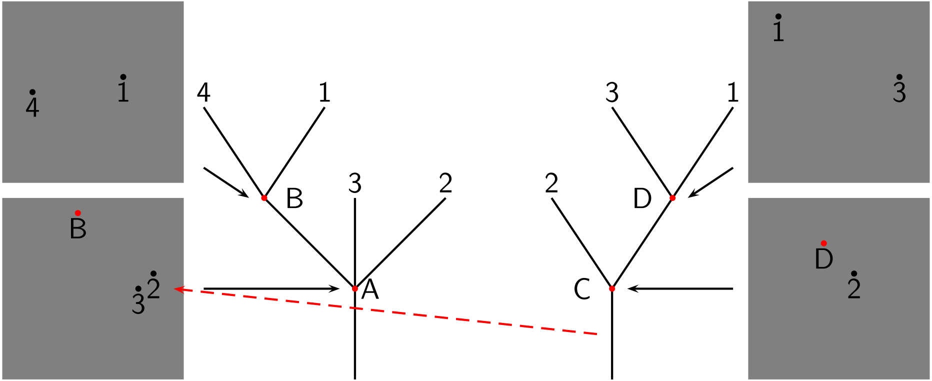

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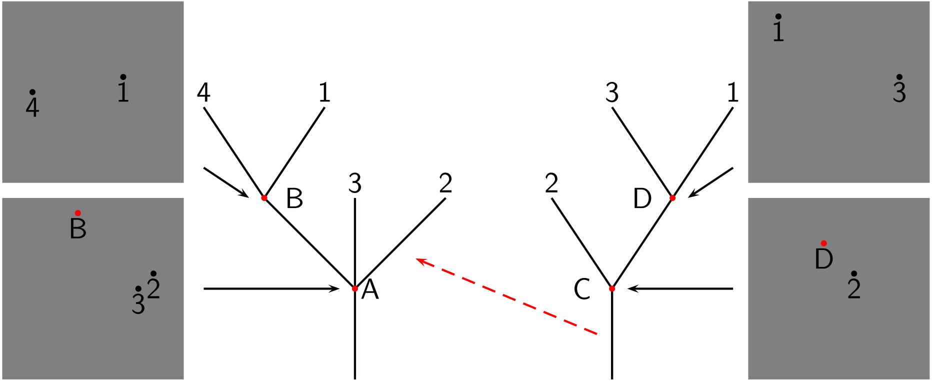

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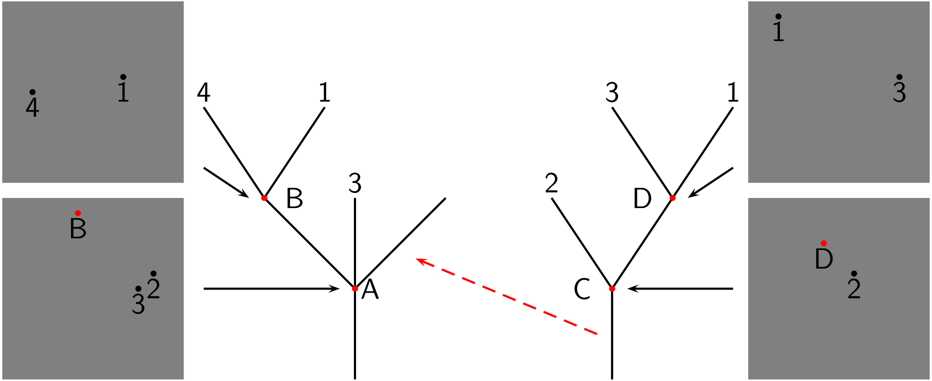

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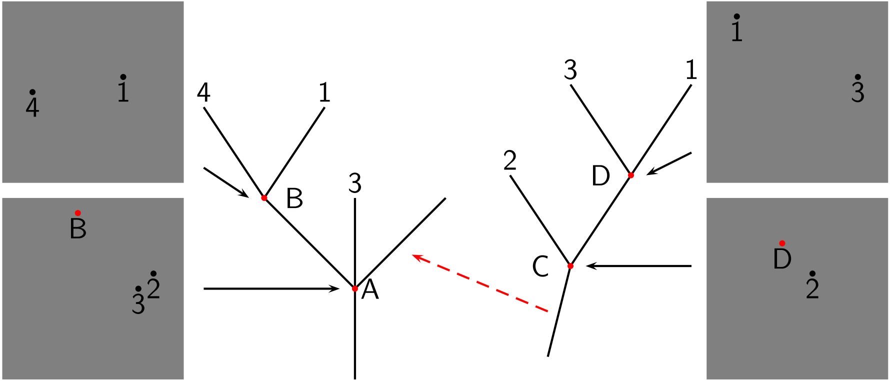

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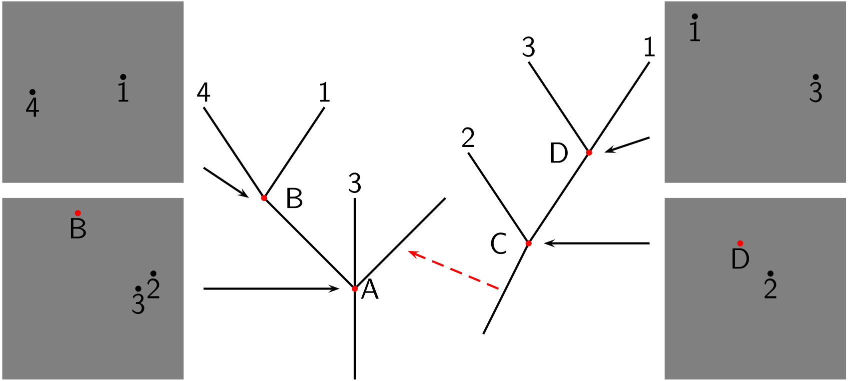

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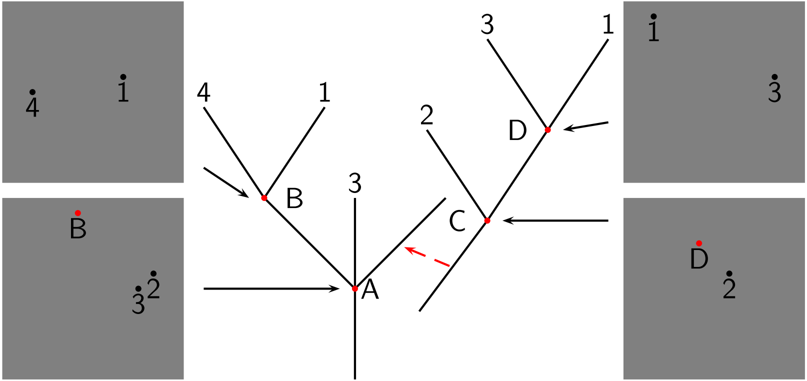

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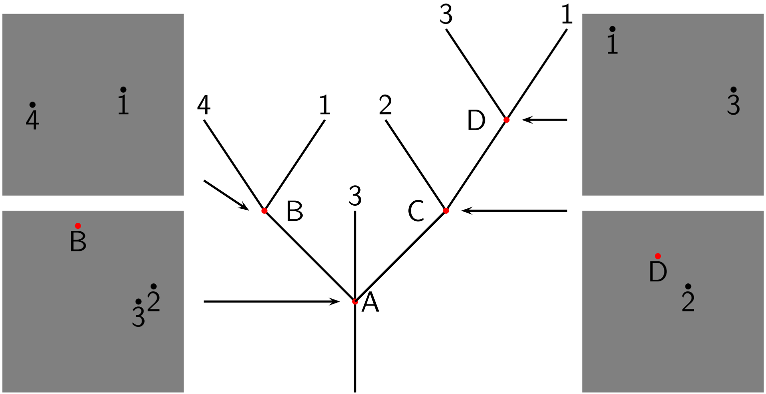

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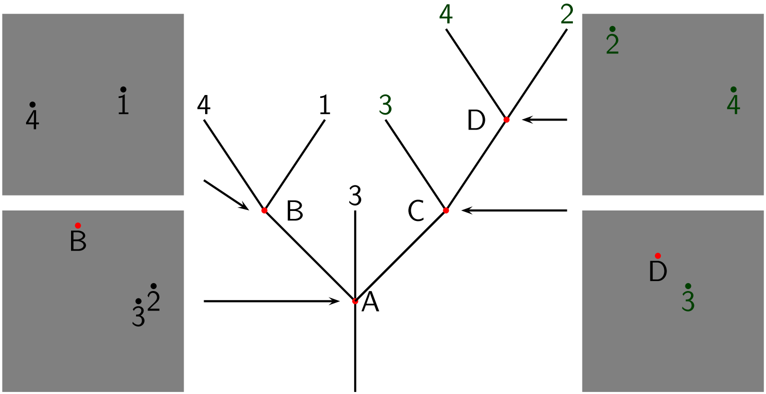

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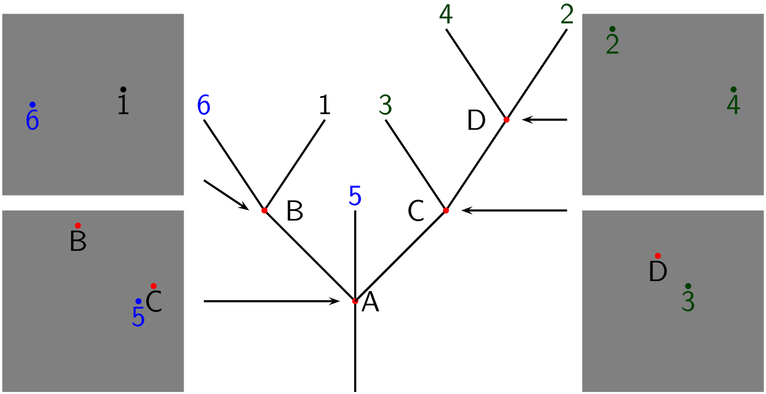

A framework for Kontsevich's statement

Conjecture/Theorem (Kontsevich, 2005)

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- Show that the pushout P_h of $rFLD \leftarrow S^1 \rightarrow tAn$ is a weak homotopy pushout
- Show that the map from the second pushout to the first pushout is a weak homotopy equivalence of operads

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Reminder

The pushout in groups is the amalgamated product $A *_B C$.

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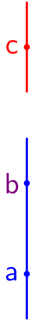

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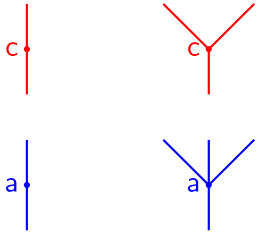

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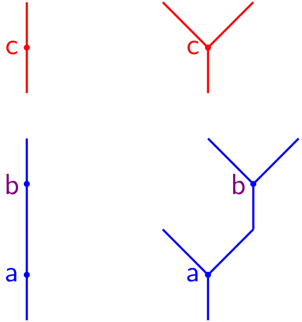

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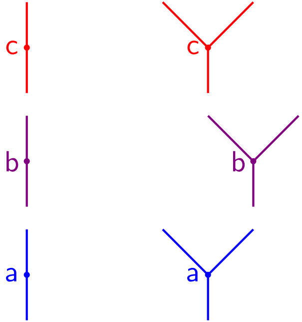

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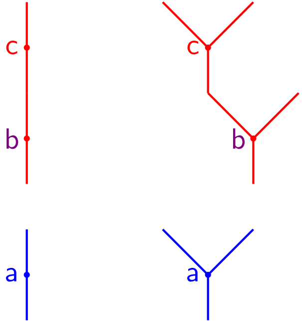

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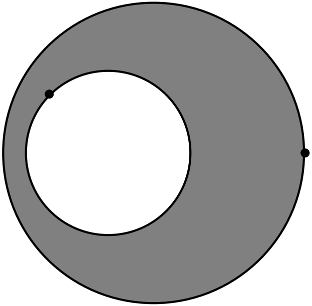

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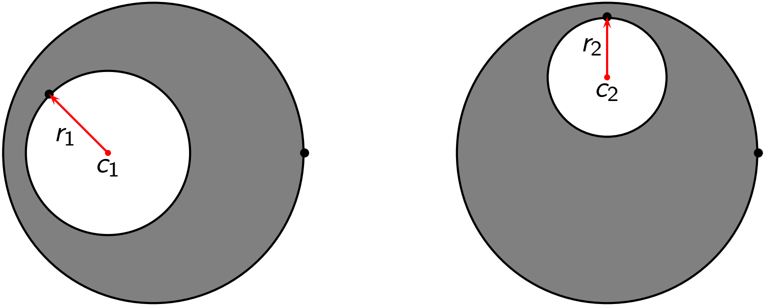

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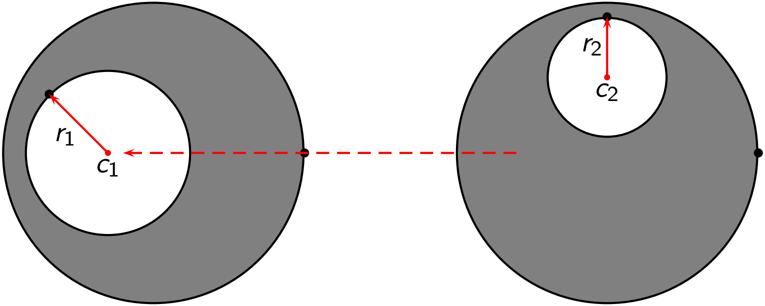

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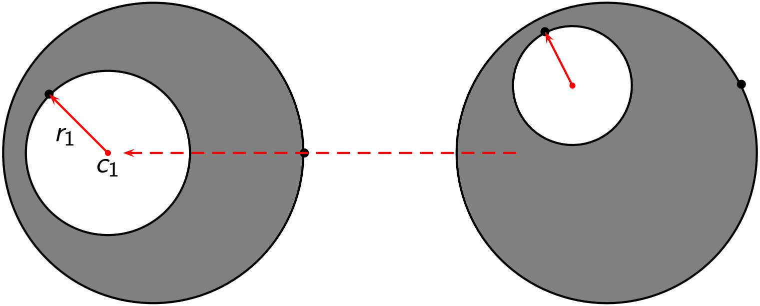

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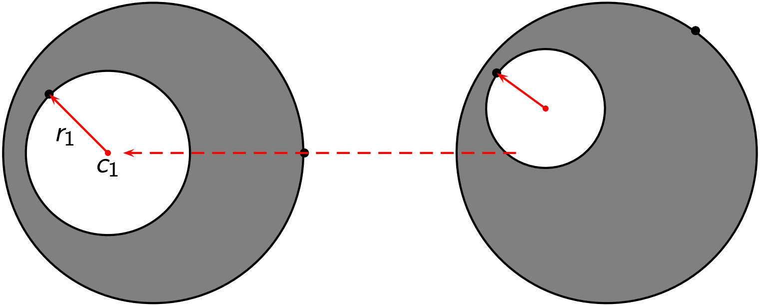

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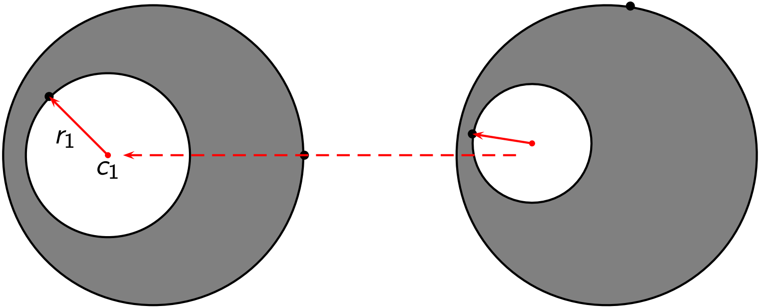

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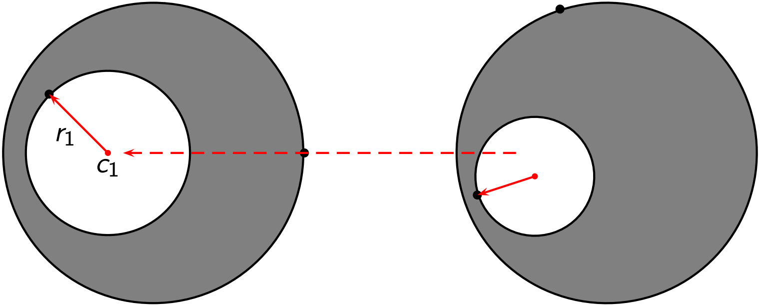

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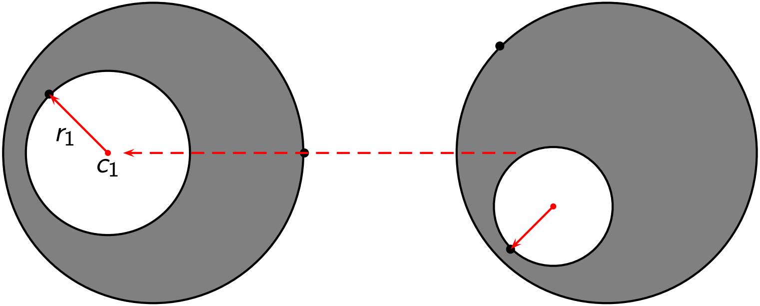

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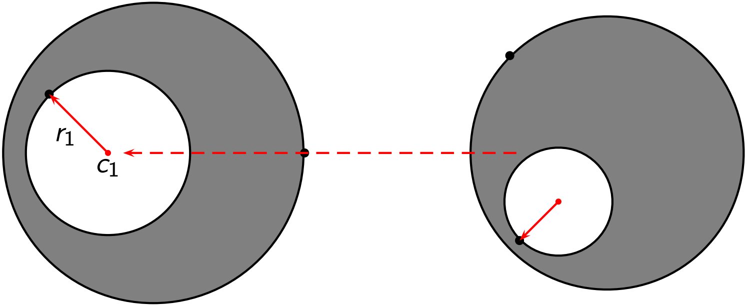

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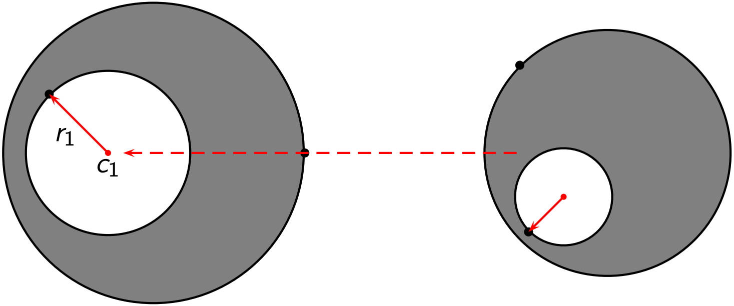

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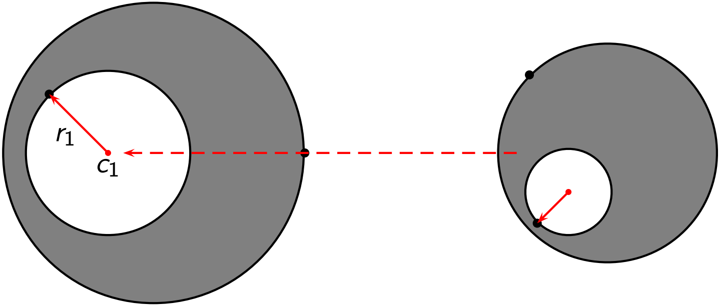

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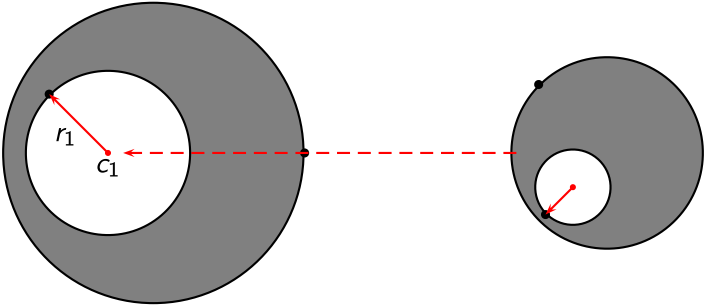

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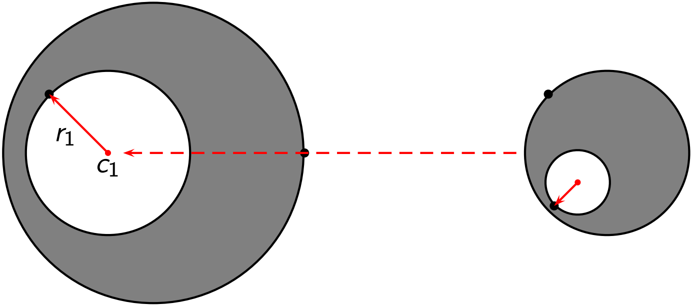

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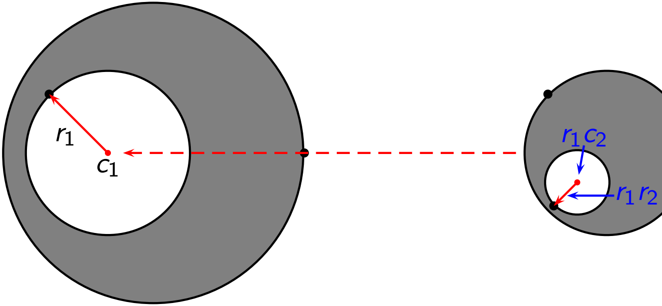

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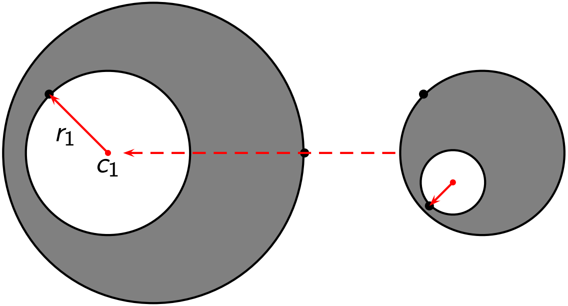

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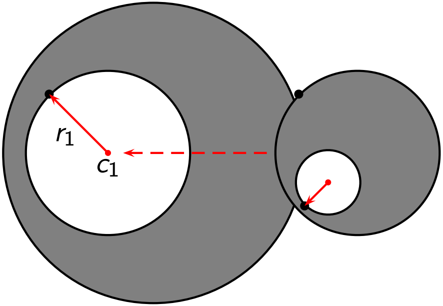

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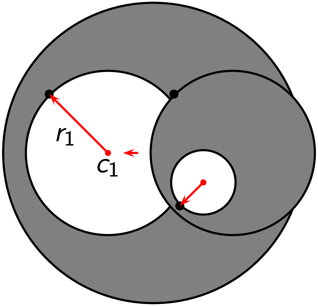

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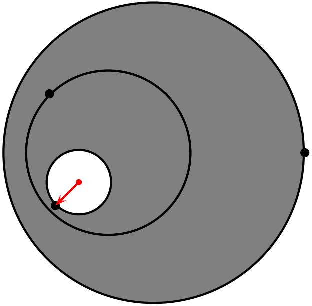

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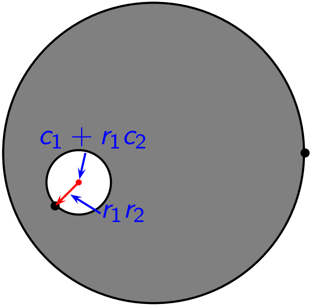

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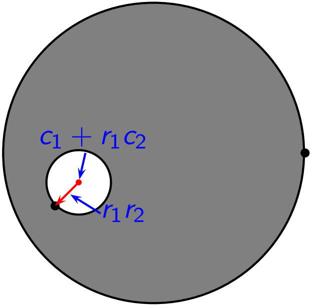

$$(c_1, r_1) \circ (c_2, r_2) = (c_1 + r_1 c_2, r_1 r_2)$$

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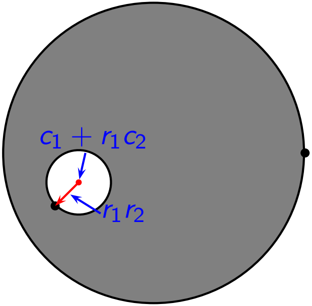

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tAn and the affine group

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$FLD(1)$ is the submonoid of $\mathrm{Aff}\,\mathbb{C}$ with $|r| + |c| \leq 1$ with this product.

Definition

$t\,\mathrm{Aff}\,\mathbb{C}$ is $\mathbb{C} \rtimes \mathbb{C}$ with this product.

Definition

tAn is the submonoid of $t\,\mathrm{Aff}\,\mathbb{C}$ with $|r| + |c| \leq 1$ with this product.

Describing $P_{\mathcal{M}}$

Describing $P_{\mathcal{M}} = FLD *_{FLD(1)} tAn$

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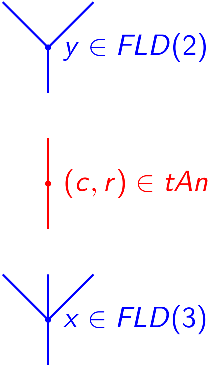

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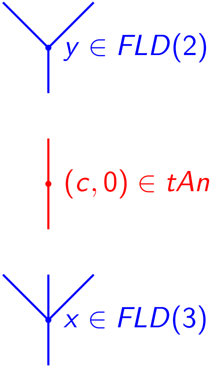

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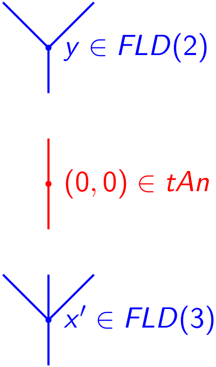

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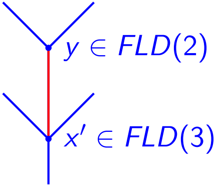

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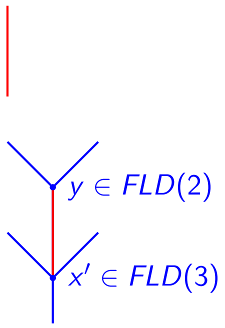

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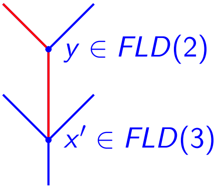

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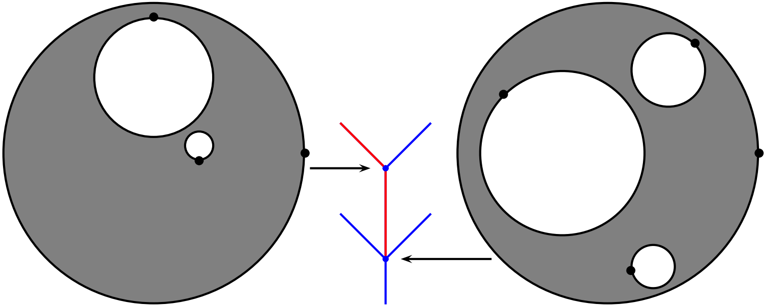

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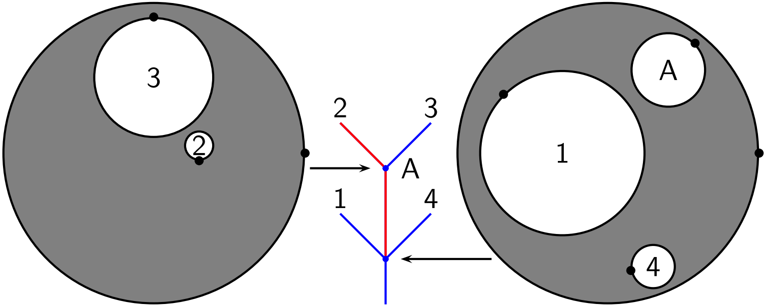

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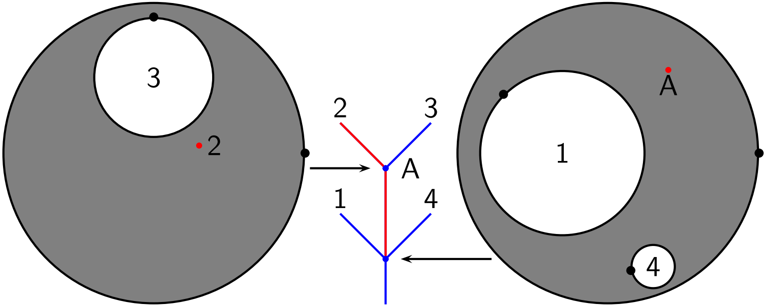

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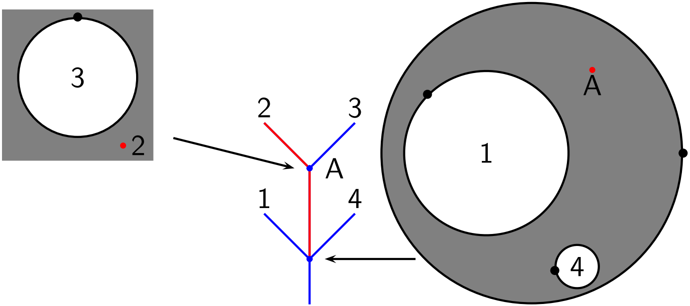

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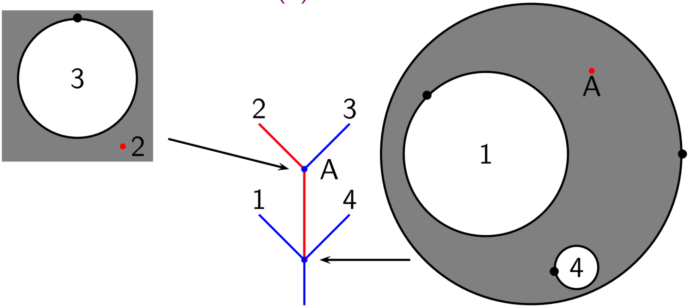

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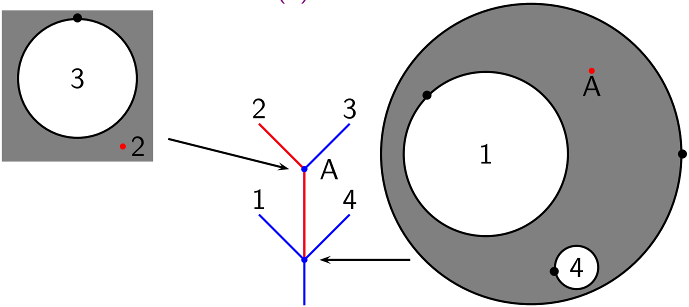
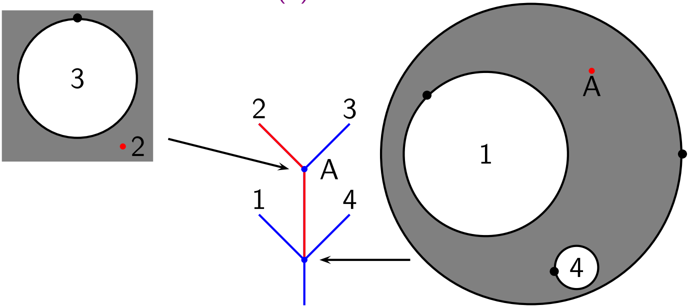


Description

$P_{\mathcal{M}}$ consists (roughly) of at least trivalent trees with some special ("blue") external edges and a label on each vertex as follows:

Describing $P_{\mathcal{M}} = FLD *_{FLD(1)} tAn$



Description

$P_{\mathcal{M}}$ consists (roughly) of at least trivalent trees with some special ("blue") external edges and a label on each vertex as follows:

a configuration of points and disks in the disk (blue outgoing edge)

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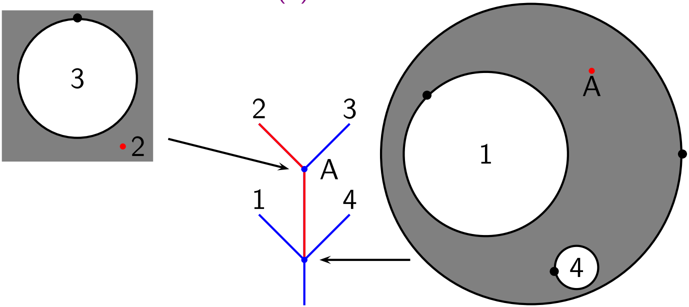

Description

$P_{\mathcal{M}}$ consists (roughly) of at least trivalent trees with some special ("blue") external edges and a label on each vertex as follows:

a configuration of points and disks in the disk (blue outgoing edge)

a configuration of points and disks in the plane up to Aff $\mathbb{C}$ (red outgoing edge)

Description

$P_{\mathcal{M}}$ consists of at least trivalent trees with some blue external edges and vertices labeled by:

- a configuration of points and disks in the disk (blue outgoing edge)
- a configuration of points and disks in the plane (red outgoing edge)
- up to $\mathrm{Aff}\,\mathbb{C}$

Description

$P_{\mathcal{M}}$ consists of at least trivalent trees with some blue external edges and vertices labeled by:

a configuration of points and disks in the disk (blue outgoing edge)

a configuration of points and disks in the plane (red outgoing edge)

up to Aff $\mathbb{C}$

Reminder

$\overline{\mathcal{M}}$ consists of at least trivalent trees with vertices labeled by:

a configuration of points in the plane

up to conformal equivalence

Description

$P_{\mathcal{M}}$ consists of at least trivalent trees with some blue external edges and vertices labeled by:

a configuration of points and disks in the disk (blue outgoing edge)
a configuration of points and disks in the plane (red outgoing edge)
up to $\mathrm{Aff}\,\mathbb{C}$

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$\overline{\mathcal{M}}$ consists of at least trivalent trees with vertices labeled by:

a configuration of points in the plane
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Description

$P_{\mathcal{M}}$ consists of at least trivalent trees with some blue external edges and vertices labeled by:

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Reminder

$\overline{\mathcal{M}}$ consists of at least trivalent trees with vertices labeled by:

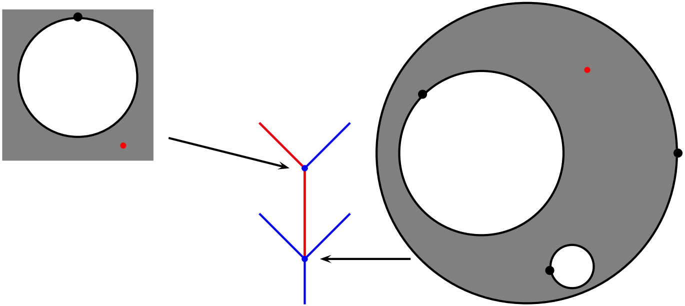
a configuration of points in the plane

up to $\mathrm{Aff}\,\mathbb{C}$

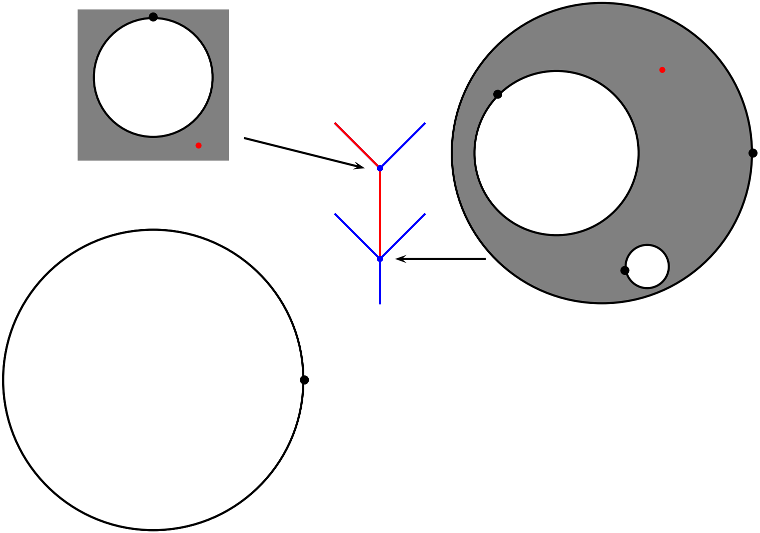
Goal

Homotope away all blue edges

Homotoping away the blue edges



Homotoping away the blue edges



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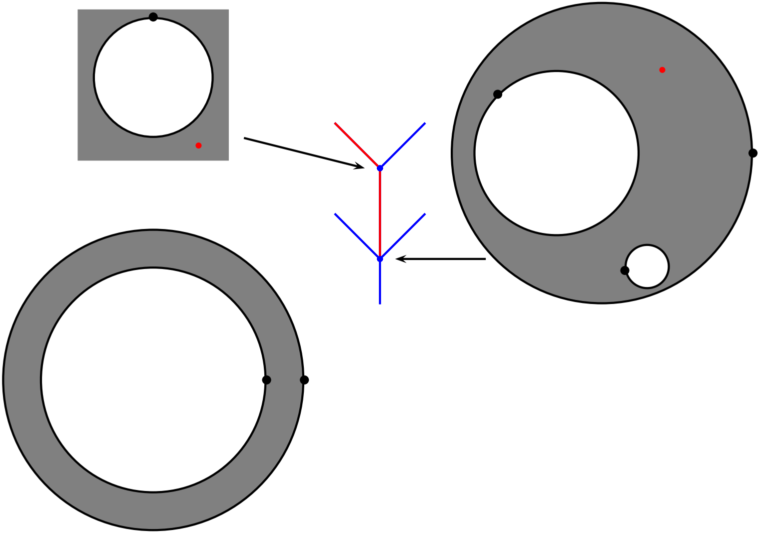

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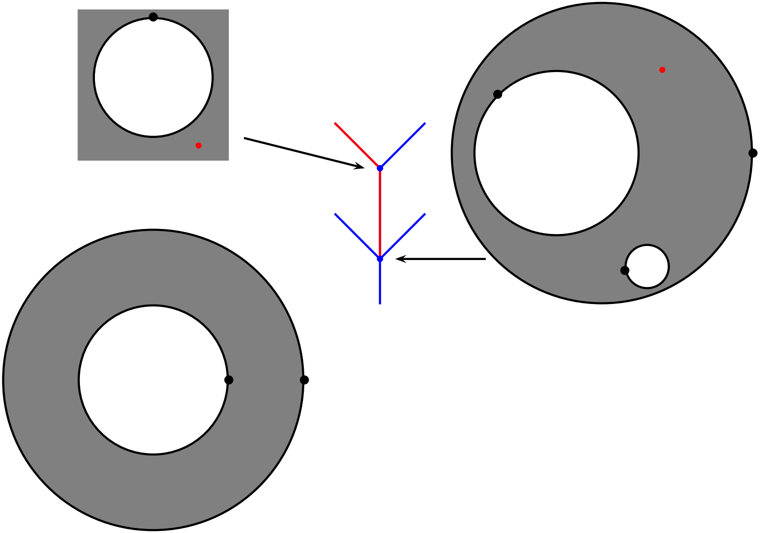

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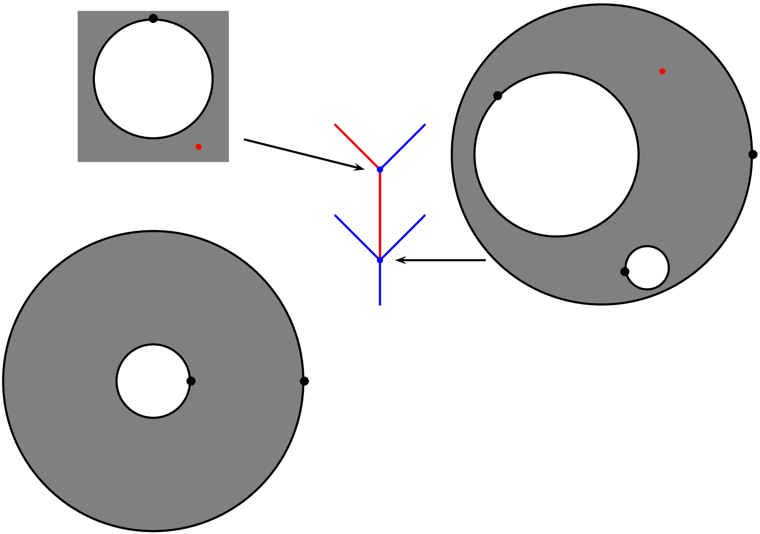

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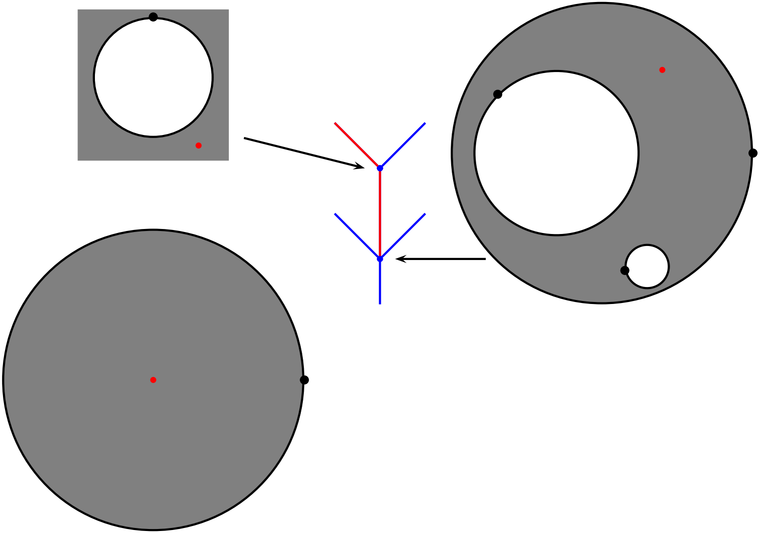

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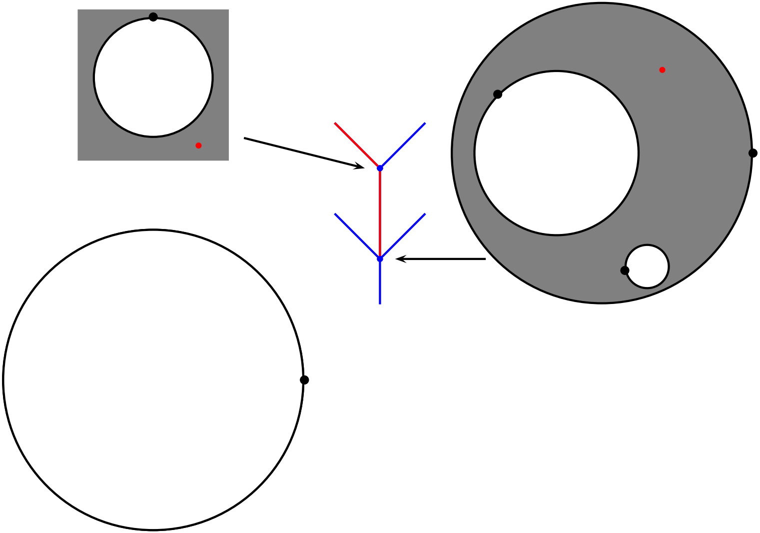

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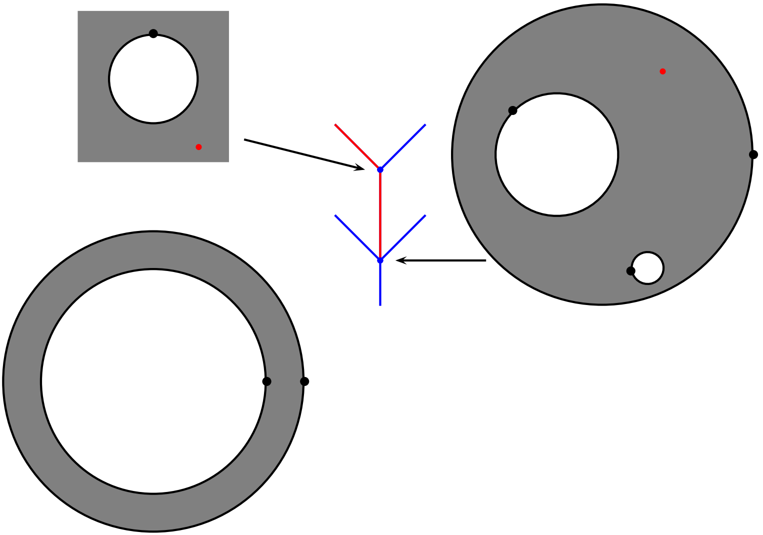

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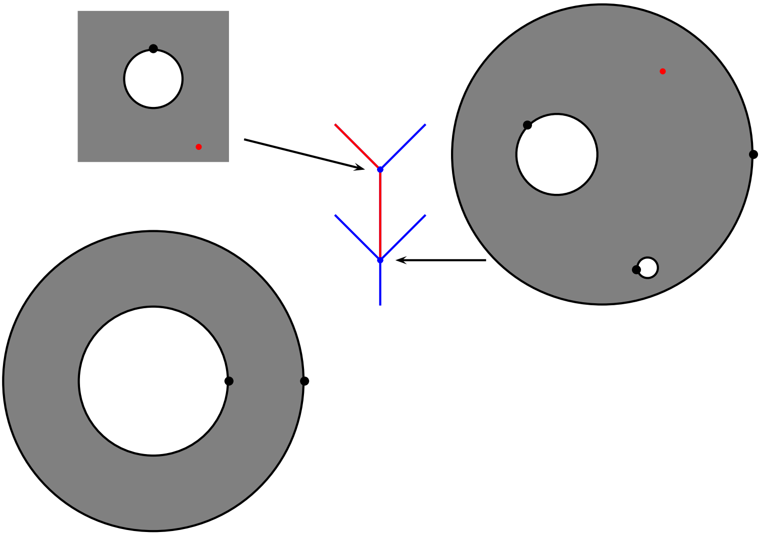

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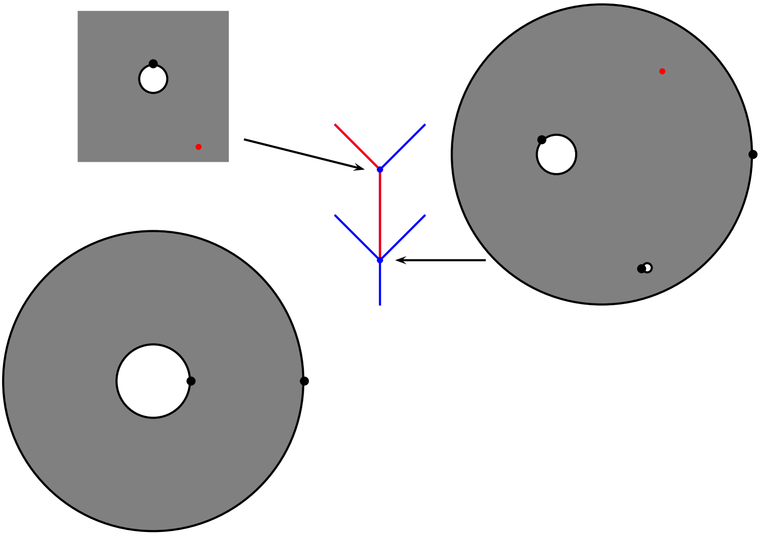

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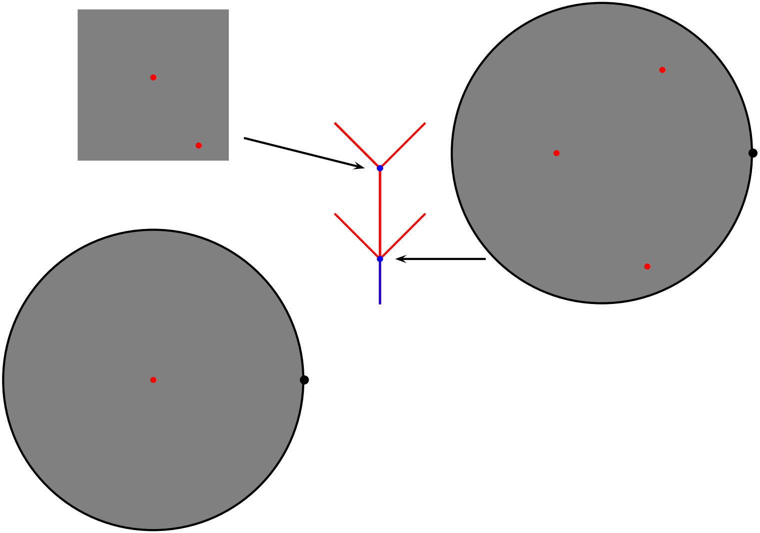

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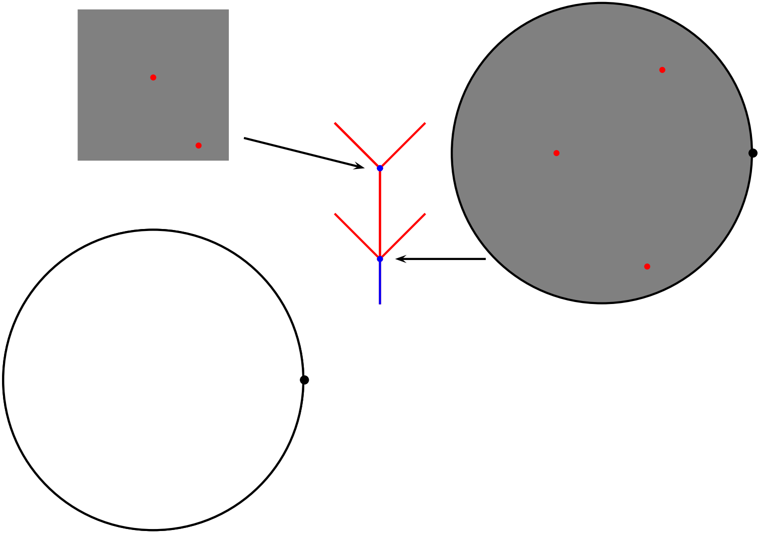

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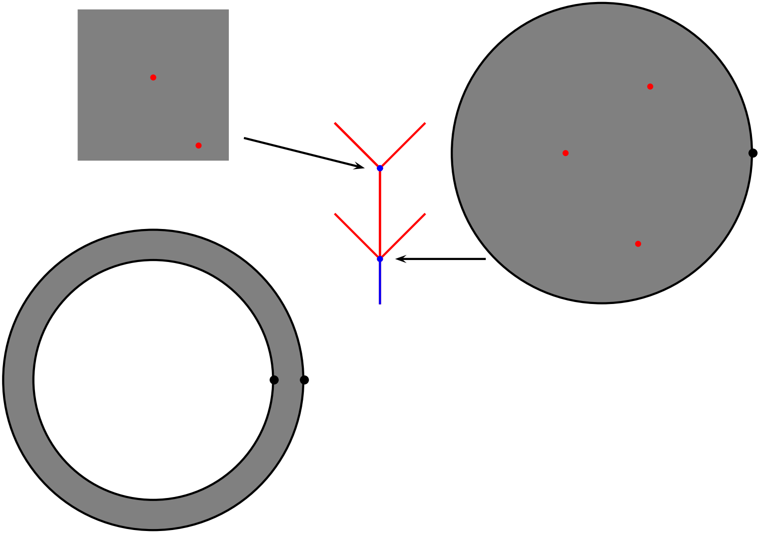

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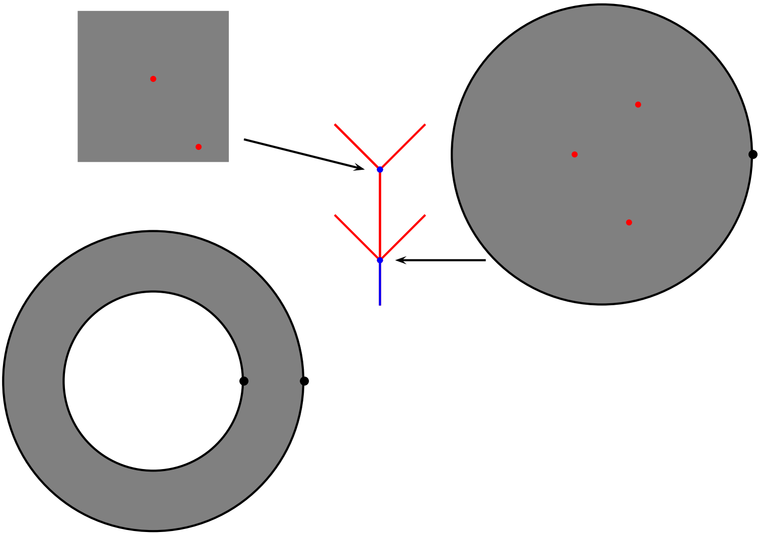

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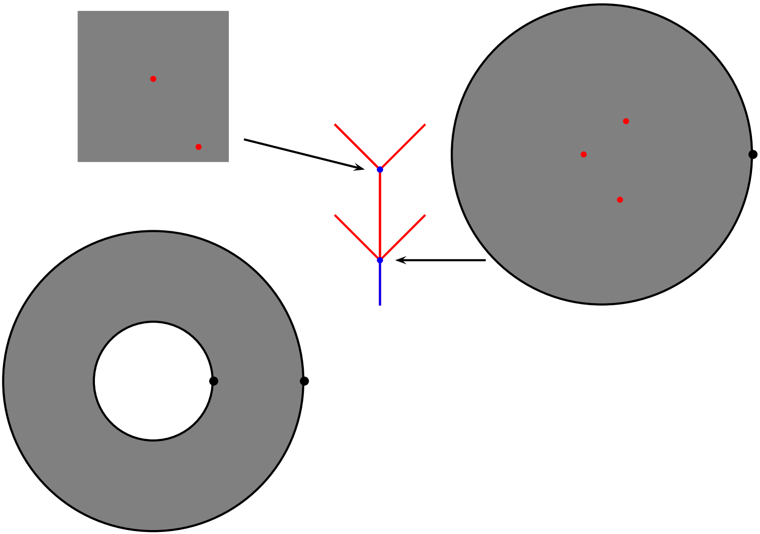

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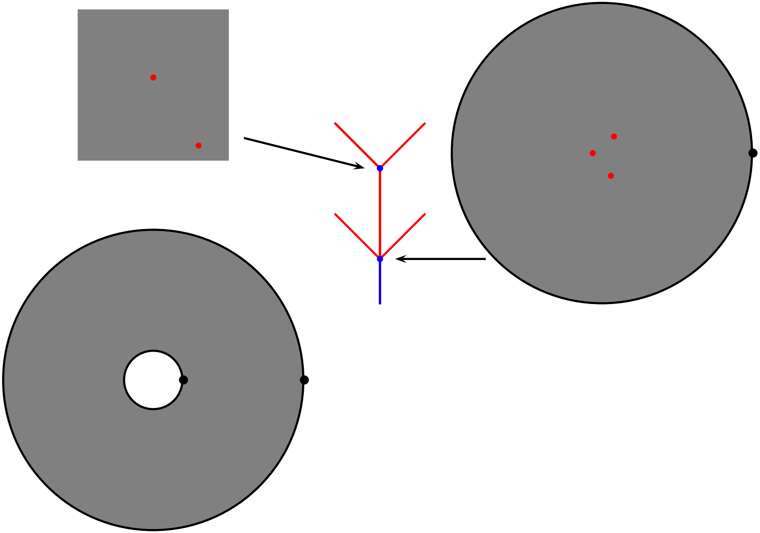

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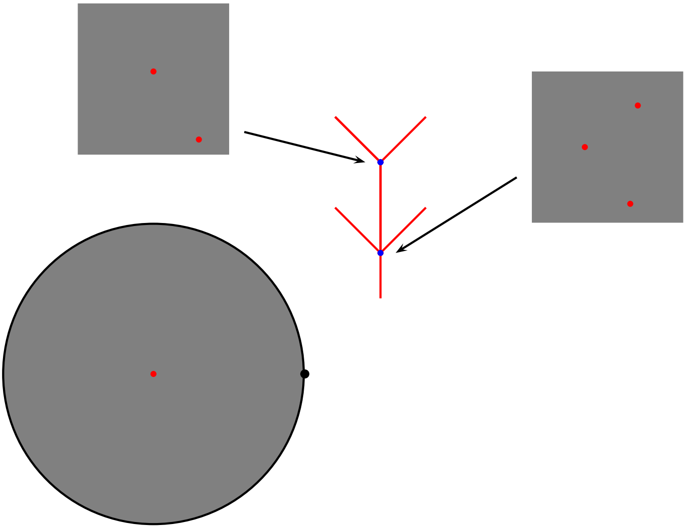

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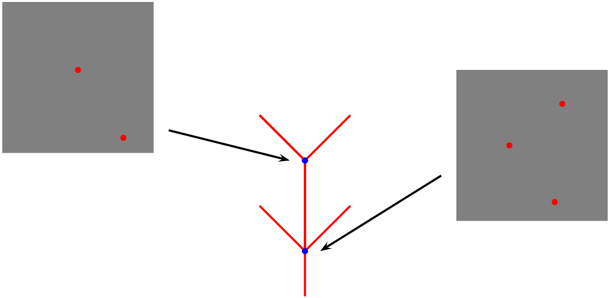
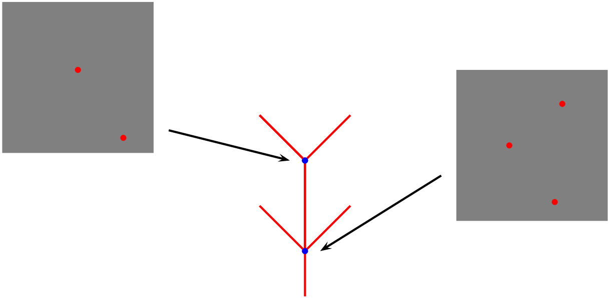


Conclusion

$\overline{\mathcal{M}}(n)$ is a deformation retract of $P_{\mathcal{M}}(n)$.

Homotoping away the blue edges



Conclusion

$\overline{\mathcal{M}}(n)$ is a deformation retract of $P_{\mathcal{M}}(n)$.
The map $\overline{\mathcal{M}} \to P_{\mathcal{M}}$ is a map of operads.

Describing P_h

Describing $P_h = rFLD *_{S^1} tAn$

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Definition

$rFLD(n) = FLD(n)$, except that

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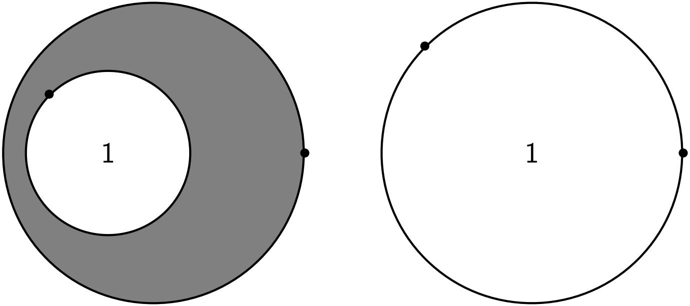
Definition

$rFLD(n) = FLD(n)$, except that $rFLD(1) = S^1$.



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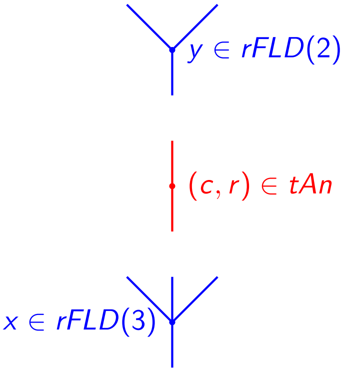

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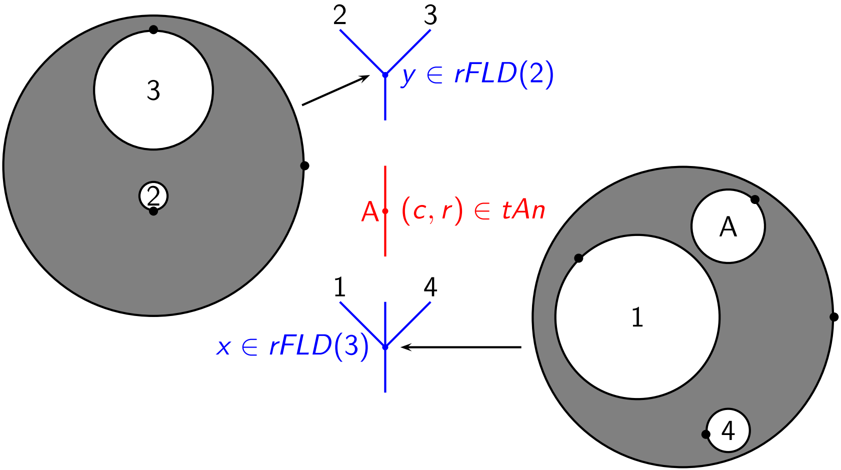

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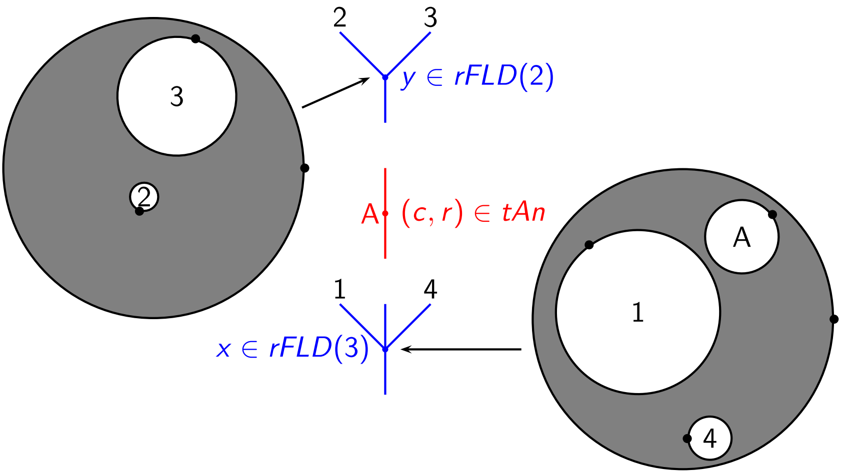

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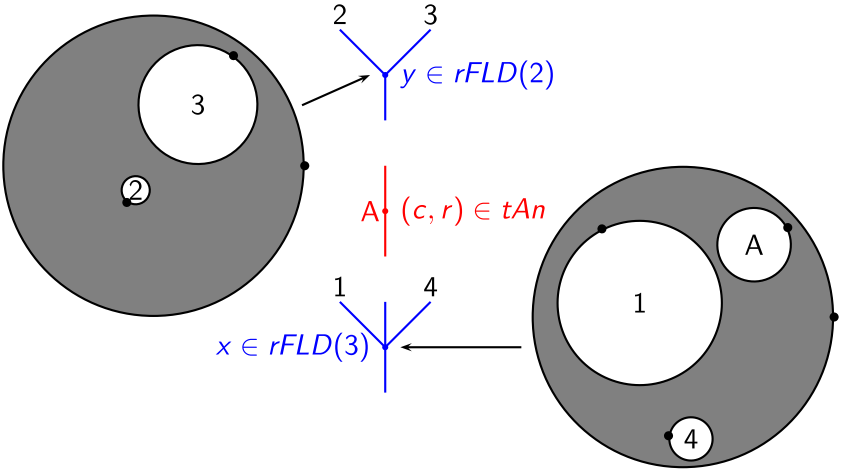

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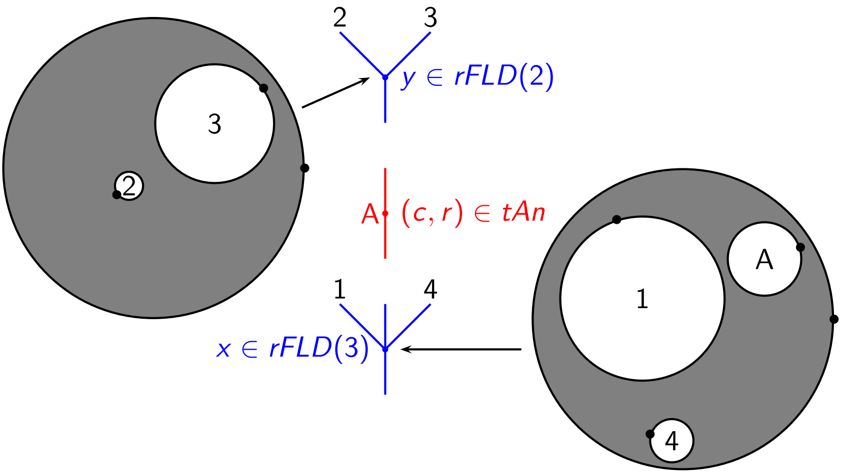

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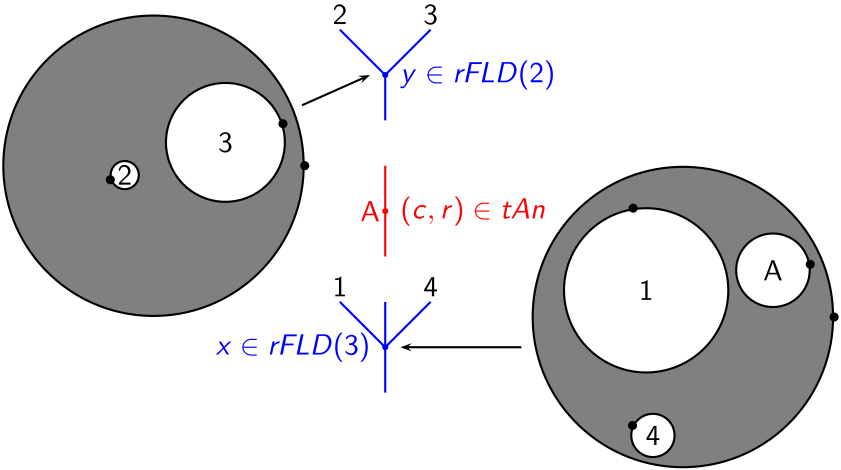

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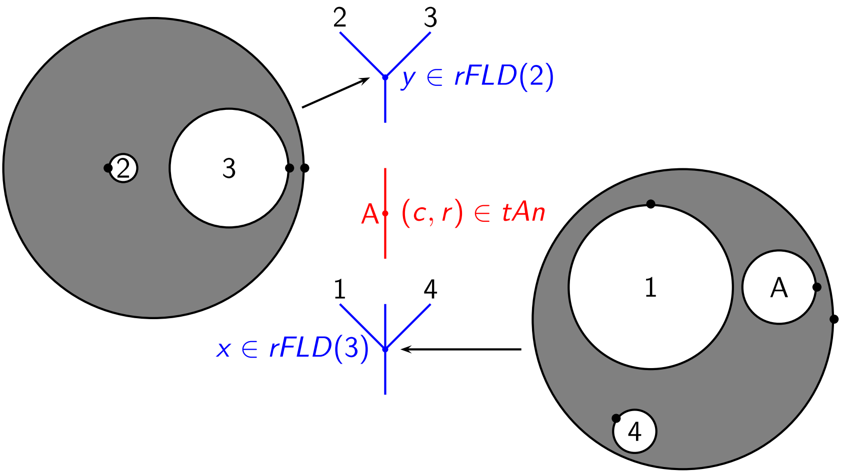

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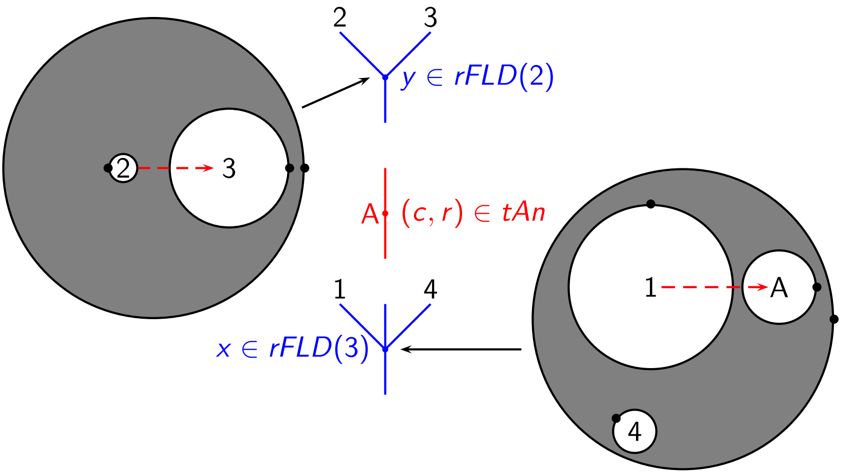

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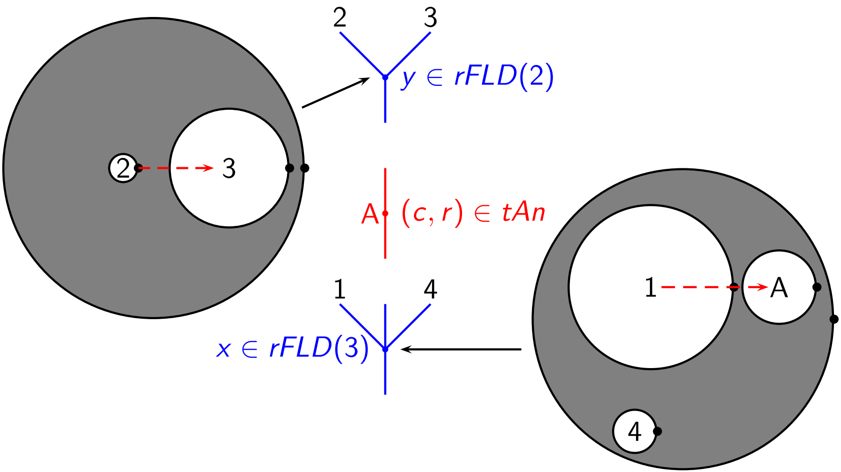

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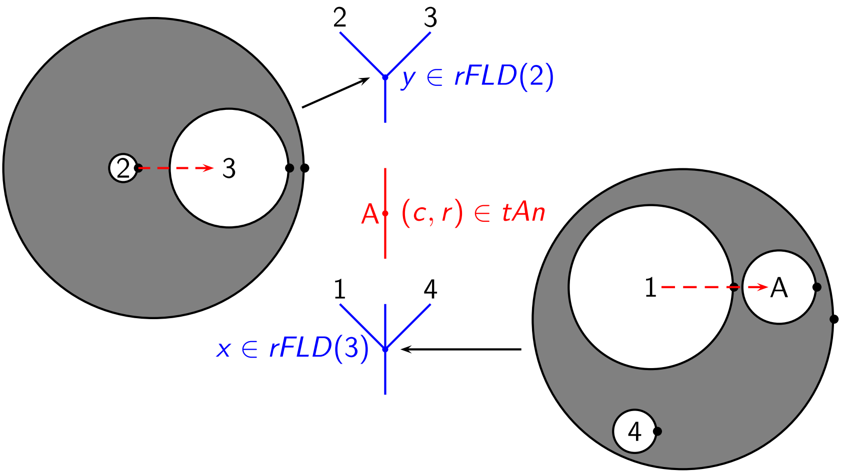

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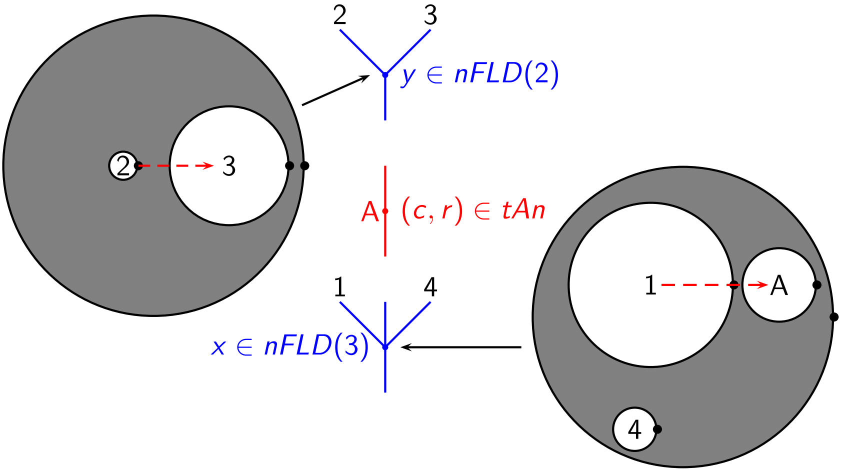

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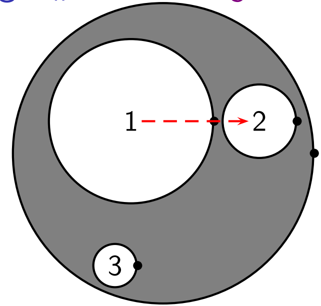

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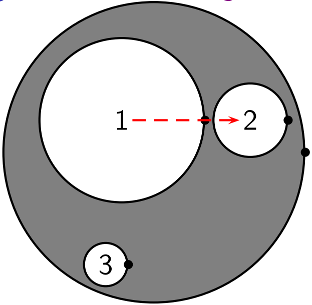

Definition

$nFLD(n)$ consists of configurations of n framed little disks so that the vector from the center of the first disk to the center of the second disk, along with all of the radius vectors, are positive reals.

Describing $P_h = rFLD *_{S^1} tAn$

Definition

$nFLD(n)$ consists of configurations of n framed little disks so that the vector from the center of the first disk to the center of the second disk, along with all of the radius vectors, are positive reals.

Observation

Every point in P_h can be realized "uniquely" as a tree with alternating bivalent and at least trivalent vertices, with markings on the bivalent vertices from tAn and on the other vertices from $nFLD$.

Showing P_h is a weak homotopy pushout

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- The framed little disks $FLD(n)$ are homeomorphic to the product $S^1 \times nFLD(n) \times (S^1)^n$

Showing P_h is a weak homotopy pushout

- The framed little disks $FLD(n)$ are homeomorphic to the product $S^1 \times nFLD(n) \times (S^1)^n$
- The inclusion of the circle into the trivializable annuli is a pointed cofibration

Showing P_h is a weak homotopy pushout

- The framed little disks $FLD(n)$ are homeomorphic to the product $S^1 \times nFLD(n) \times (S^1)^n$
- The inclusion of the circle into the trivializable annuli is a pointed cofibration
- Weak equivalences and cofibrations of spaces interact nicely with coproducts and products

Taking stock

Taking stock

$$P_{\mathcal{M}} \underset{\sim}{\overset{\sim}{\rightleftarrows}} \overline{\mathcal{M}}$$

Taking stock

$$P_h \qquad P_{\mathcal{M}} \underset{\sim}{\overset{\sim}{\leftrightarrows}} \overline{\mathcal{M}}$$

Taking stock

$$P_h \longrightarrow P_{\mathcal{M}} \underset{\sim}{\overset{\sim}{\rightleftarrows}} \overline{\mathcal{M}}$$

Taking stock

$$P_h \longrightarrow P_{\mathcal{M}} \underset{\sim}{\overset{\sim}{\rightleftarrows}} \overline{\mathcal{M}}, \qquad \tau : P_h \dashrightarrow \overline{\mathcal{M}}$$

Taking stock

$$P_h \longrightarrow P_{\mathcal{M}} \underset{\sim}{\overset{\sim}{\leftrightarrows}} \overline{\mathcal{M}} \qquad \tau : P_h \dashrightarrow \overline{\mathcal{M}}$$

A criterion to show that a map is a weak equivalence

A map of spaces f is a weak equivalence if its range has an open cover, closed under finite intersection, and the restriction of f to each element of the cover is a weak equivalence.

Taking stock

$$\begin{array}{ccccc} & & \tau & & \\ P_h & \longrightarrow & P_{\mathcal{M}} & \underset{\sim}{\overset{\sim}{\rightleftarrows}} & \overline{\mathcal{M}} \end{array}$$

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U

Taking stock

$$P_h \longrightarrow P_{\mathcal{M}} \underset{\sim}{\overset{\sim}{\rightleftarrows}} \overline{\mathcal{M}}, \qquad \tau : P_h \dashrightarrow \overline{\mathcal{M}}$$

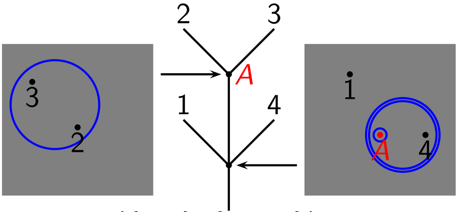
A criterion to show that a map is a weak equivalence

A map of spaces f is a weak equivalence if its range has an open cover, closed under finite intersection, and the restriction of f to each element of the cover is a weak equivalence.

$$\tau^{-1}(U) \longrightarrow U$$

Describing the local weak equivalence

U consists of configurations that can be simultaneously separated into certain partitions.



A point in $U(\{2,3\};\{2,3,4\})$

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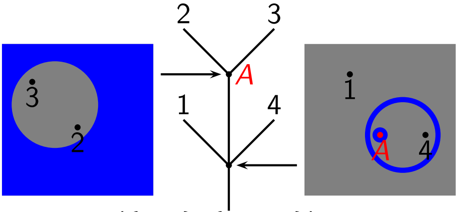

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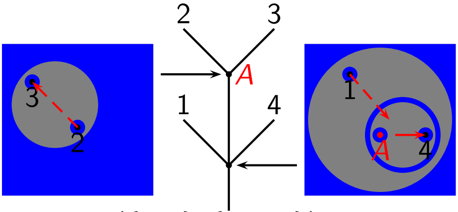


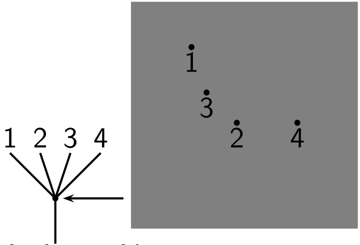
A point in $U(\{2,3\};\{2,3,4\})$ or its preimage

Describing the local weak equivalence

U consists of configurations that can be simultaneously separated into certain partitions.

Simplifying assumptions

- Restrict to $\mathcal{M}$ inside $\overline{\mathcal{M}}$



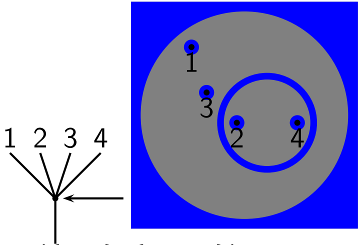
A point in $U(\{2,3\};\{2,3,4\})$

Describing the local weak equivalence

U consists of configurations that can be simultaneously separated into certain partitions.

Simplifying assumptions

- Restrict to $\mathcal{M}$ inside $\overline{\mathcal{M}}$
- Discard inappropriate separations in the preimage



A point in the preimage of $U(\{2,3\};\{2,3,4\})$

Describing the local weak equivalence

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Simplifying assumptions

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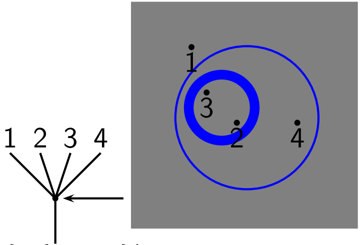

A point in $U(\{2,3\};\{2,3,4\})$

Describing the local weak equivalence

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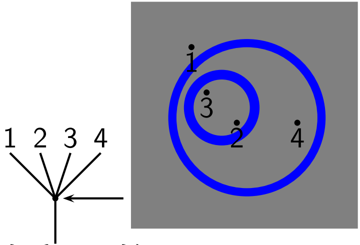
- Discard inappropriate and ambiguous separations in the preimage
- This cover is not closed under finite intersection



A point in $U(\{2,3\};\{2,3,4\})$

Justifying the restriction to $\mathcal{M}$

Problem

It is not enough to achieve a weak equivalence on neighborhoods over $\mathcal{M}$; these need to have appropriate limiting behavior at the boundary.

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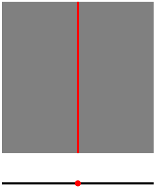

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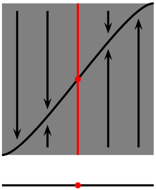
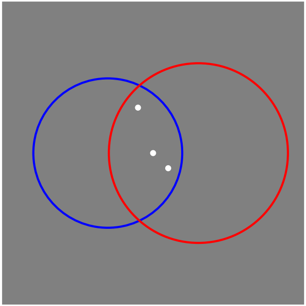


Solution

Find a deformation retraction that is fixed fiberwise and that has appropriate limiting behavior.

Picturing the retraction



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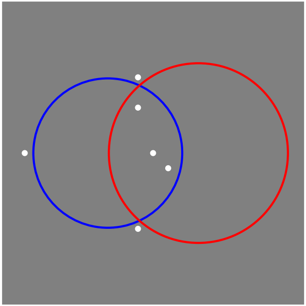

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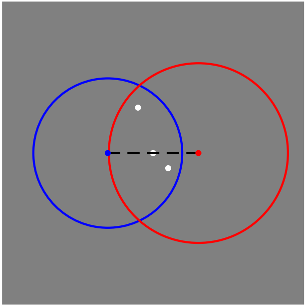

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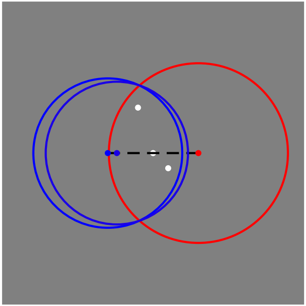

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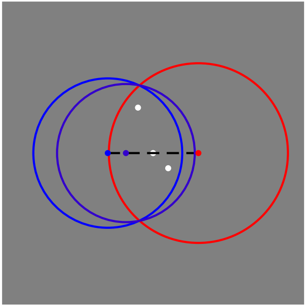

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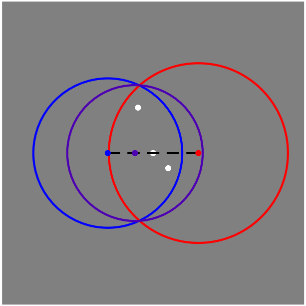

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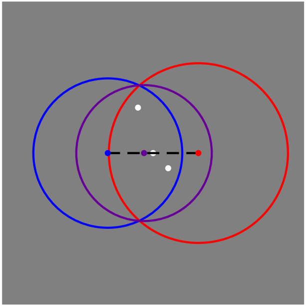

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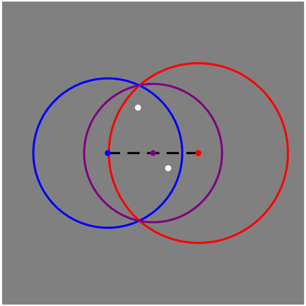

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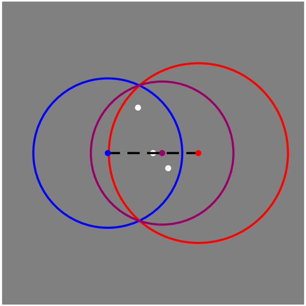

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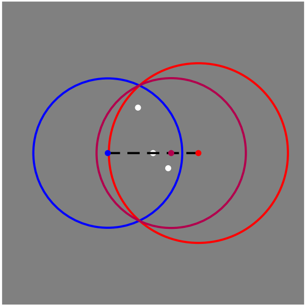

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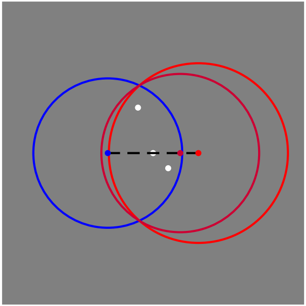

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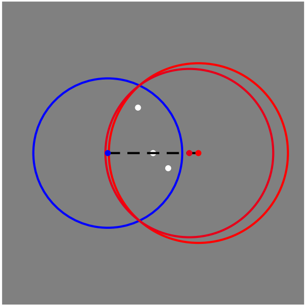

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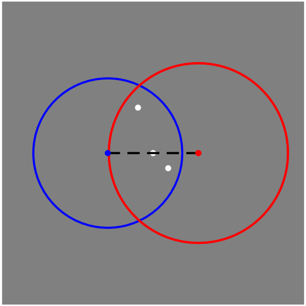
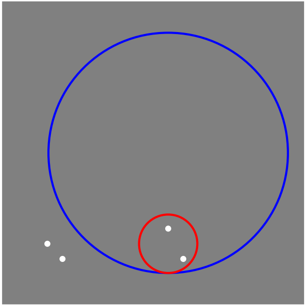
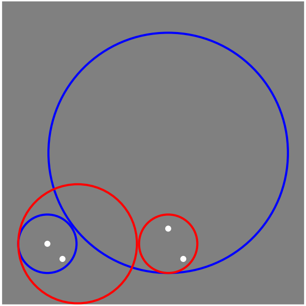


Issues with the straight line homotopy



- Making sure disjoint pairs stay disjoint

Issues with the straight line homotopy



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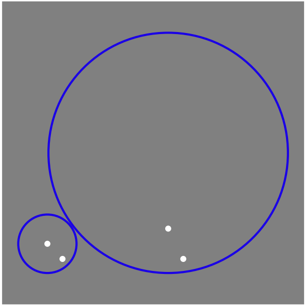

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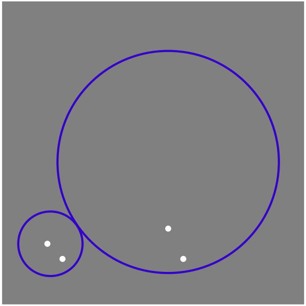

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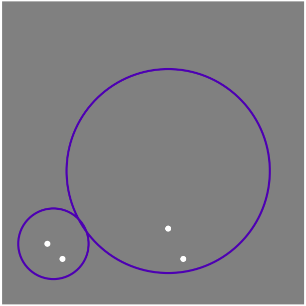

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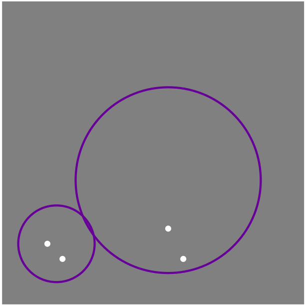

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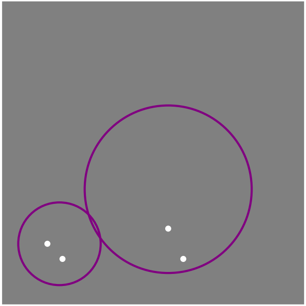

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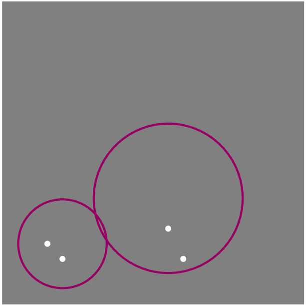

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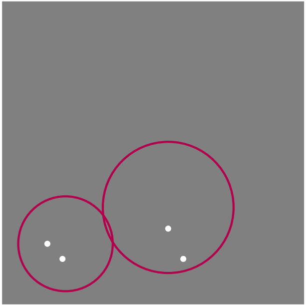

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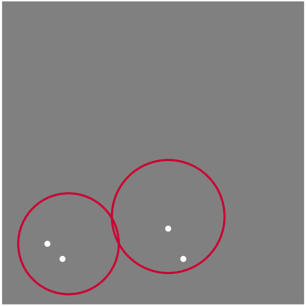

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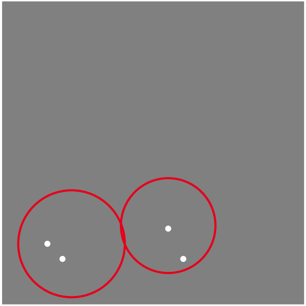

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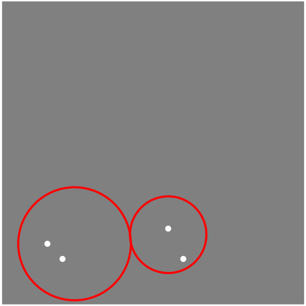

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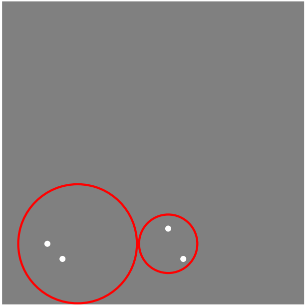
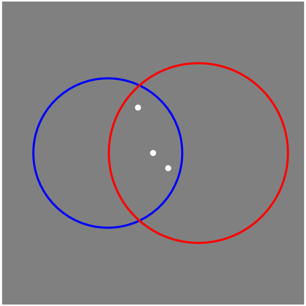


- Making sure disjoint pairs stay disjoint
- Making sure nested pairs stay nested

A solution to the problem



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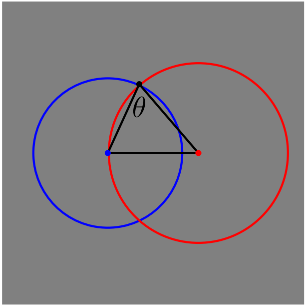

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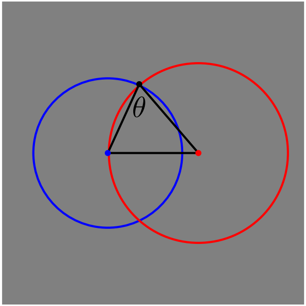

Homotope through θ rather than along the distance.

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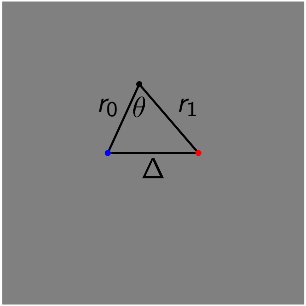

Homotope through θ rather than along the distance.

$$\theta = \arccos\left(\frac{r_0^2 + r_1^2 - \Delta^2}{2 r_0 r_1}\right)$$

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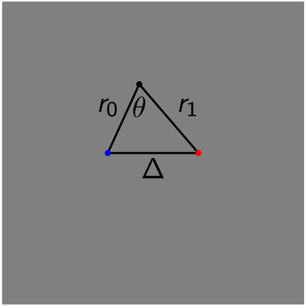

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This formula makes sense as long as the radii are nonzero. θ is either in $[0, \pi)$ or is a positive imaginary number.

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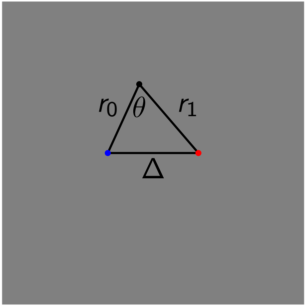

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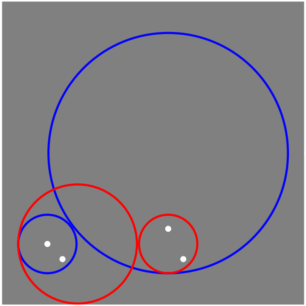
$$c_t = \frac{c_1 r_0 \sin(t\theta) + c_0 r_1 \sin((1-t)\theta)}{r_0 \sin(t\theta) + r_1 \sin((1-t)\theta)}, \qquad r_t = \frac{r_0 r_1 \sin(\theta)}{r_0 \sin(t\theta) + r_1 \sin((1-t)\theta)}$$

Some evidence



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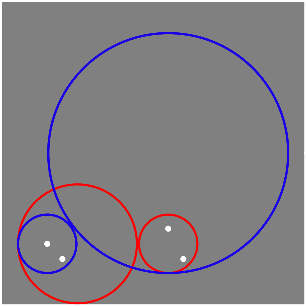

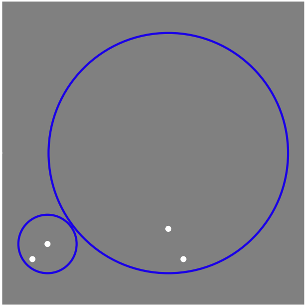

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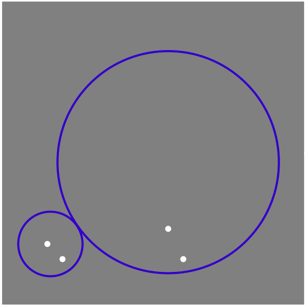 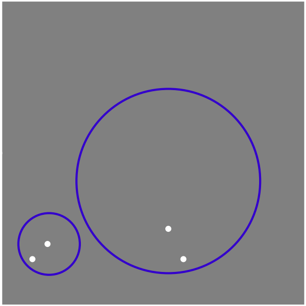

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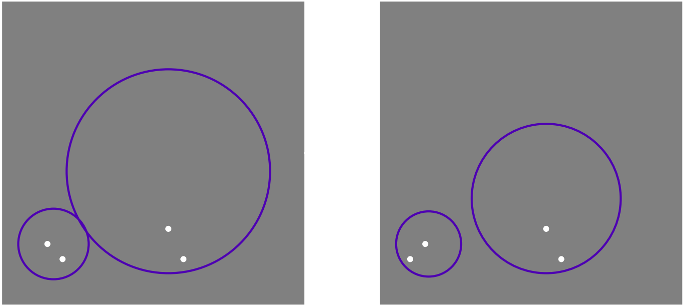

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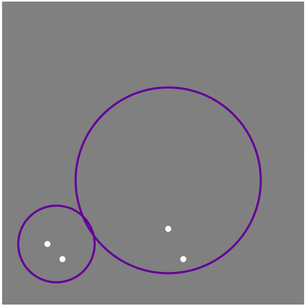

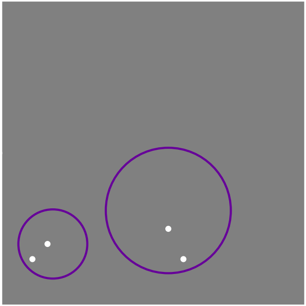

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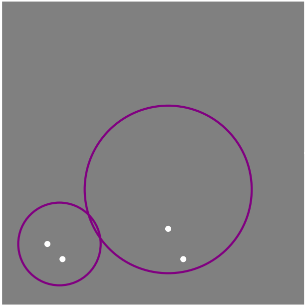

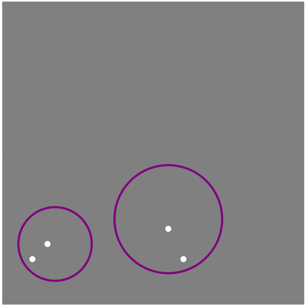

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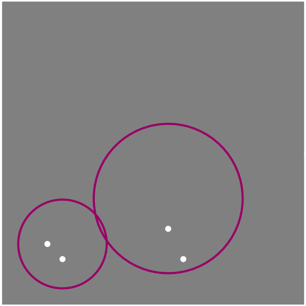

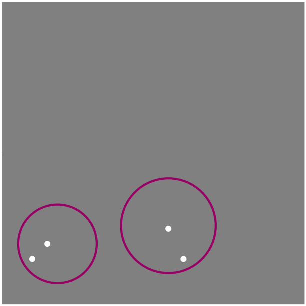

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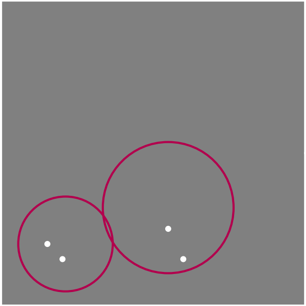

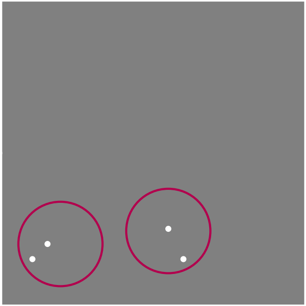

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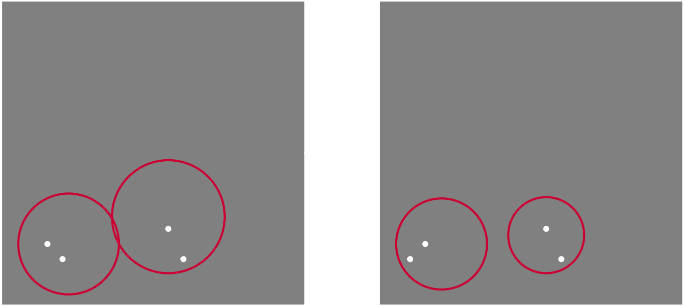

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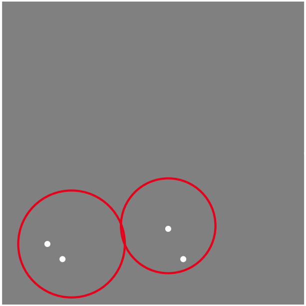

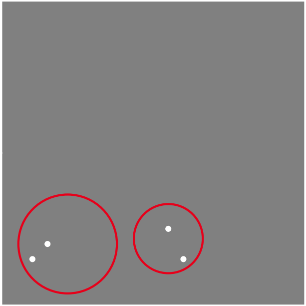

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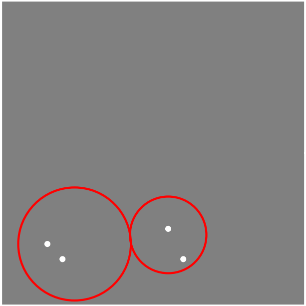

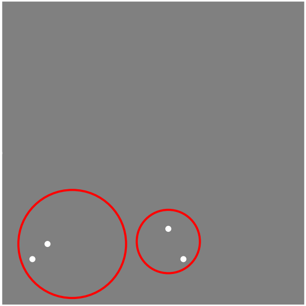

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