Edge stabilization in graph configuration spaces

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(joint with Byunghee An and Ben Knudsen)

https://arxiv.org/abs/1708.02351 https://arxiv.org/abs/1806.05585

IBS-CGP

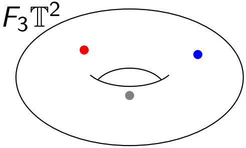
June 25, 2018

Configuration spaces

Definition (Configuration spaces)

Let X be a space. The *ordered configuration space* $F_k(X)$ of X is the space of k-tuples of distinct points in X:

$$F_k(X) := \{(x_1, \ldots, x_k) \in X^k | x_i \neq x_j \text{ for } i \neq j\}.$$



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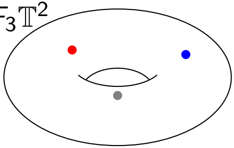
The *unordered configuration space* $B_k(X)$ is the quotient of $F_k(X)$ by the symmetric group action.

$$B_k(X) := F_k(X)/S_k$$

$F_3\mathbb{T}^2$



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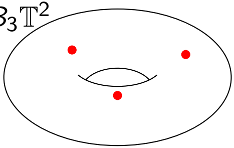
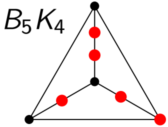


Configuration spaces of graphs

This talk is about the unordered configuration space of a graph Γ.



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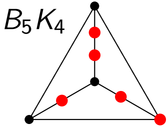

Theorem (Ghrist; Abrams)

Let Γ be a connected graph. The space $B_k(\Gamma)$ is a $K(\pi, 1)$ space.

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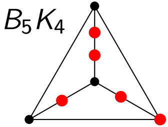


Theorem (Ghrist; Abrams)

Let Γ be a connected graph. The space $B_k(\Gamma)$ is a $K(\pi, 1)$ space.

Motivating question

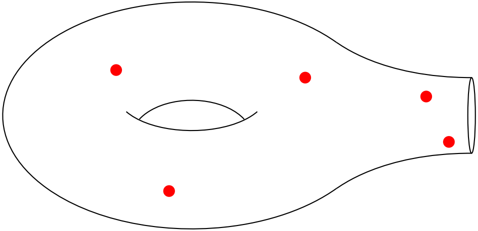
Let Γ be a graph. Calculate the bigraded groups $H_i(B_k(\Gamma))$.

(bigrading by *homological degree* and *cardinality*).

Stabilization phenomena

Stabilization at boundary components

Let M be a manifold with a boundary component ∂.



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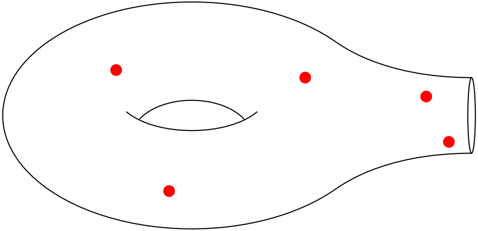
Let M be a manifold with a boundary component ∂.
There is a ∂-*stabilization* operation from $B_k(M)$ to $B_{k+1}(M)$.



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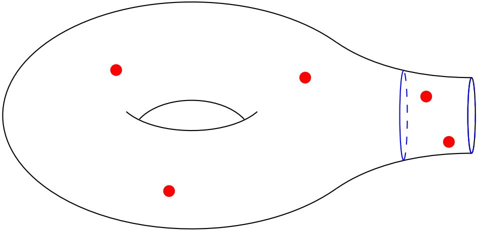

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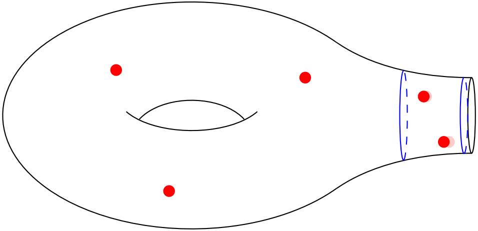

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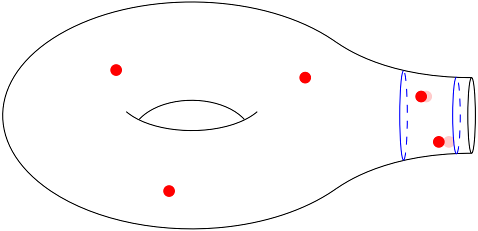

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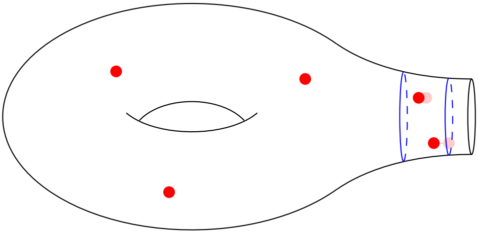

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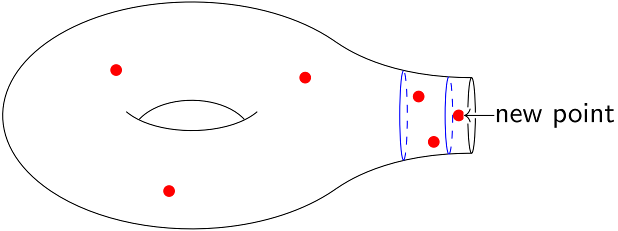

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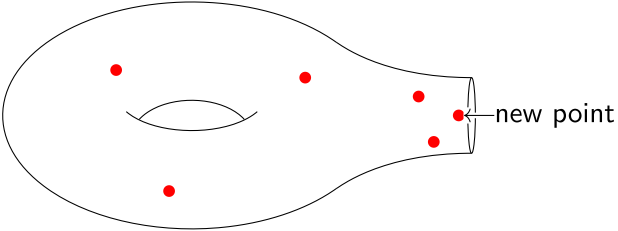

We stabilize at ∂ by deforming a cylindrical end near ∂ inwards from the boundary and adding a new point.

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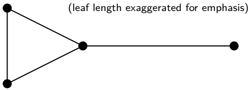
We can always stabilize at a cylindrical end in any space.
For example, we can stabilize at a leaf of a graph.



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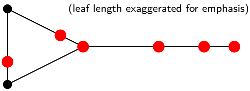

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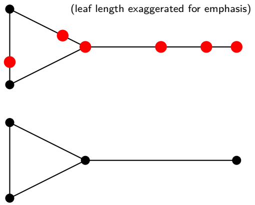

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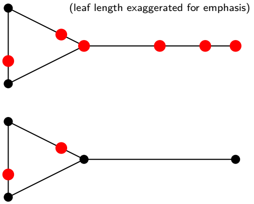

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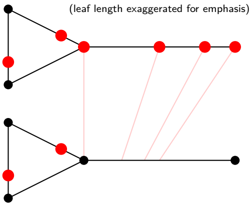

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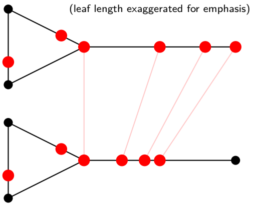

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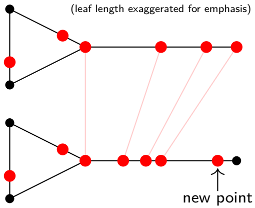

$$B_6(\Gamma) \longrightarrow B_7(\Gamma)$$

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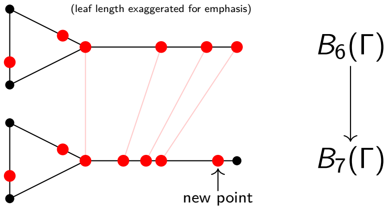

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Idea: use midpoints between configuration points in the leaf.

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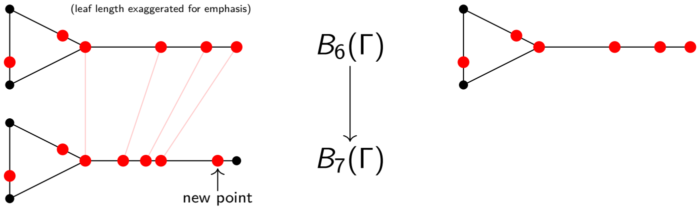

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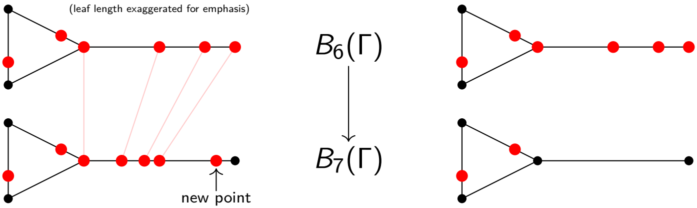

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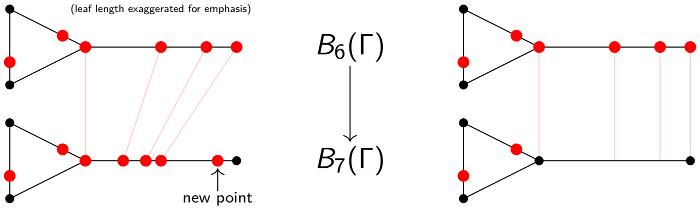

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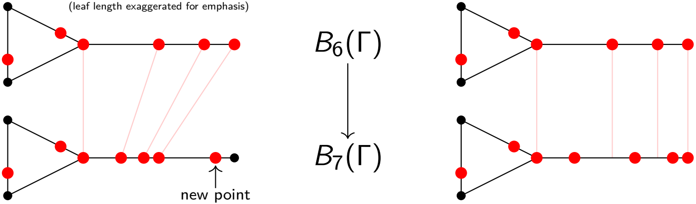

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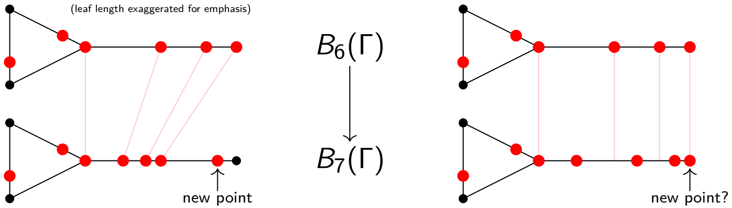

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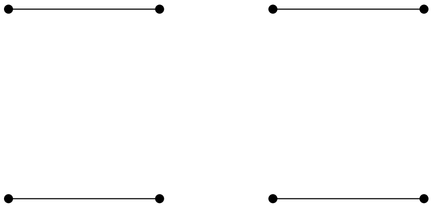
Midpoint stabilization

Key observation

The midpoint parameterization is (basically) insensitive to the presence of configuration points at the vertices.



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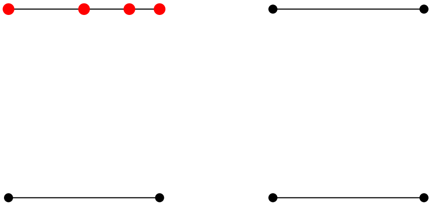

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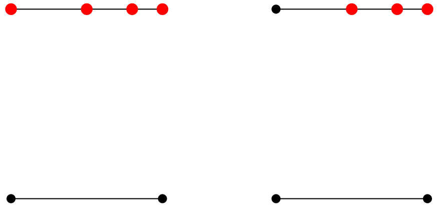

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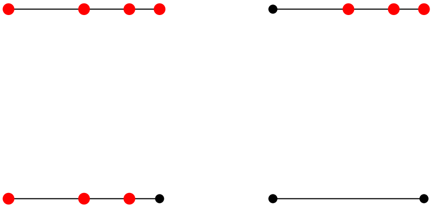

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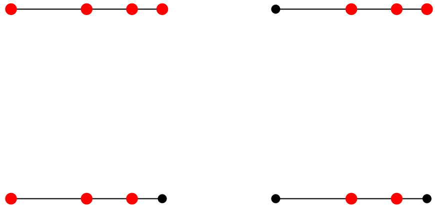

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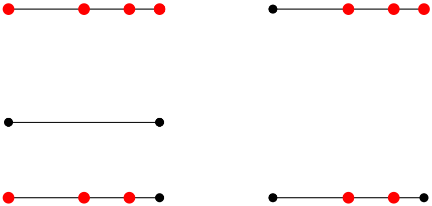

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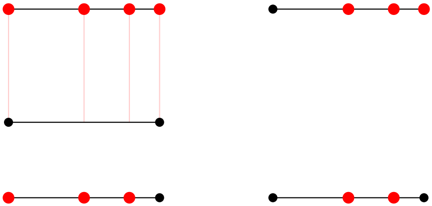

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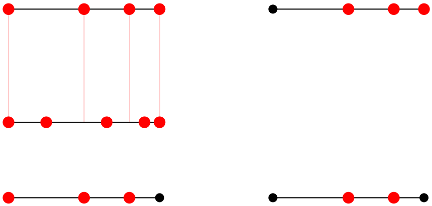

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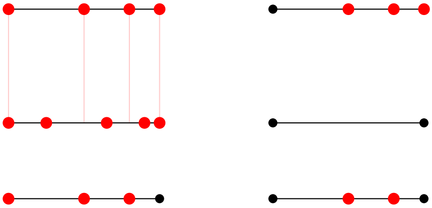

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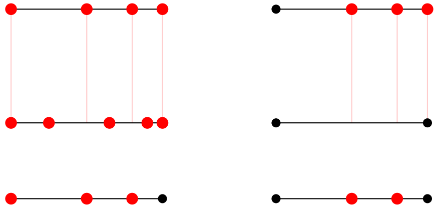

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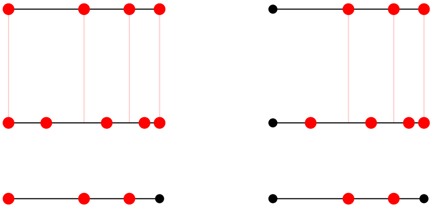

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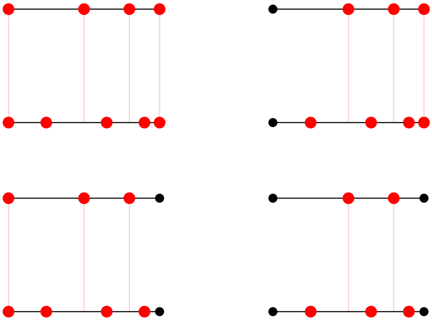
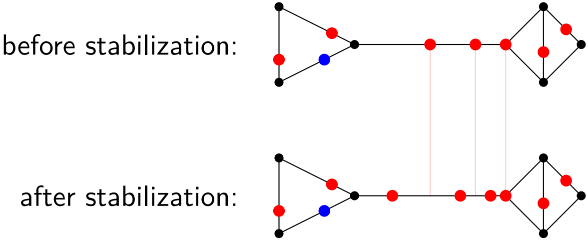


Edge stabilization

Consequence

We can stabilize at any edge, not just at leaves.



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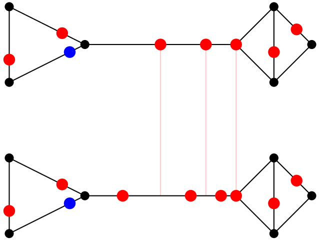

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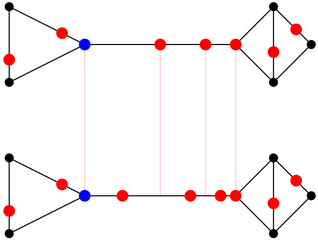

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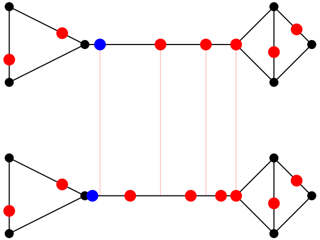

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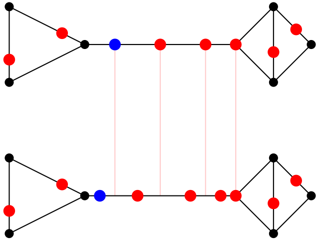

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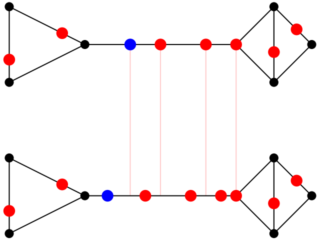

Edge stabilization: framework

Fix a graph Γ with edges E.

Proposition (An–D.–Knudsen)

The singular chains $C_*(B_*(\Gamma))$ are a differential bigraded $\mathbb{Z}[E]$-module.

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Proposition (Ramos; An–D.–Knudsen)

The homology groups $H_*(B_*(\Gamma))$ are (naturally) a bigraded $\mathbb{Z}[E]$-module.

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Finite generation has the following consequence.

Corollary

Over a field, for any i, $\dim H_i(B_k(\Gamma))$ is eventually polynomial in k.

Growth rate of the homology

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Theorem (Ramos; An–D.–Knudsen)

The polynomial degree is equal to a certain connectivity invariant of Γ.

The invariant is roughly the maximum number of connected components of the complement of i vertices in Γ.

Formality over the stabilization ring

The chain module is richer than the homology module.

Theorem (An–D.–Knudsen)

The chains $C_*(B_*(\Gamma))$ are formal as a $\mathbb{Z}[E]$-module if and only if Γ is a disjoint union of connected small graphs.

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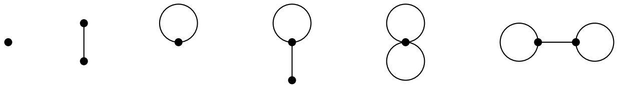

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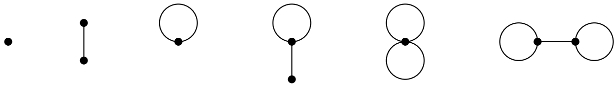

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Non-formality means the homotopy type of the homology module is not the same as the homotopy type of the chain module.

A local chain model

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Construction-Definition: $S(v)$

Consider the free bigraded $\mathbb{Z}[e_1, \ldots, e_n]$-module $S(v)$ on the generators:

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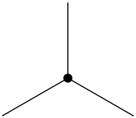

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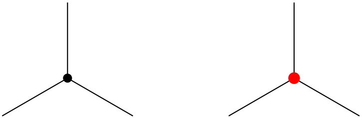

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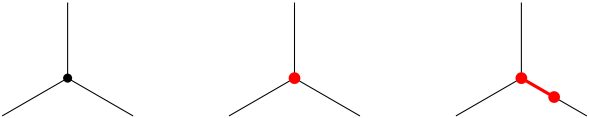

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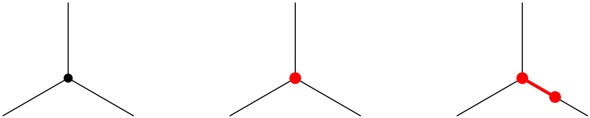

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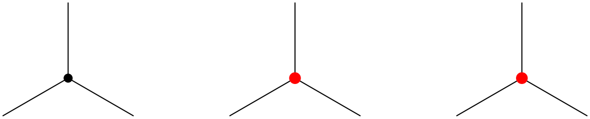

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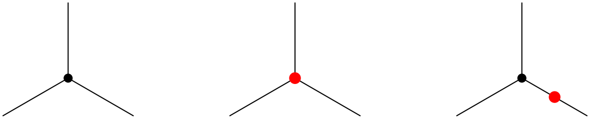

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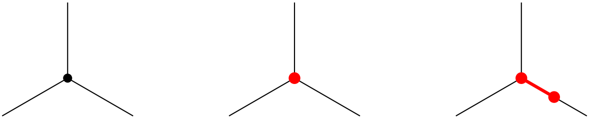

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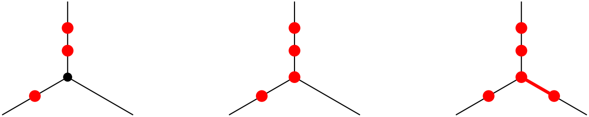

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Theorem (Świątkowski; Lütgehetmann; Chettih–Lütgehetmann; An–D.–Knudsen)

There is a finitely generated bigraded differential $\mathbb{Z}[E]$*-module* $S_{*,*}(\Gamma)$ *weakly equivalent to* $C_*(B_*(\Gamma))$.

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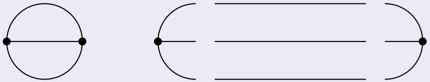
Example



$$S_{*,*}(\Gamma) \cong \quad S(v) \quad \otimes_{\mathbb{Z}[x_1,x_2,x_3]} \quad S(w)$$

Star classes

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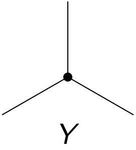

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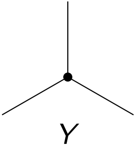

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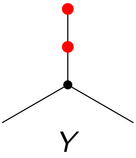

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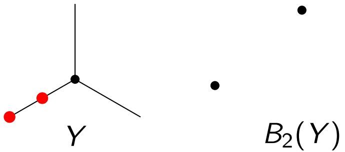

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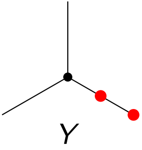

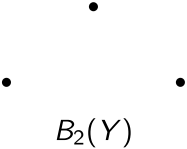

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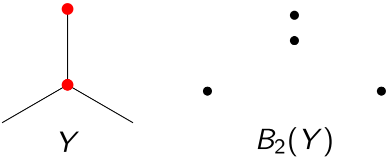

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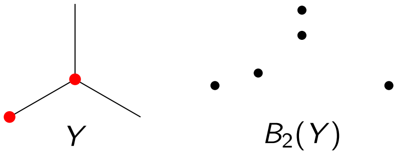

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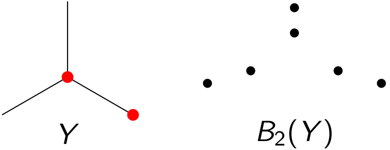

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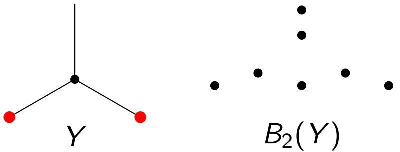

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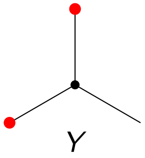

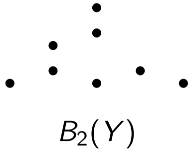

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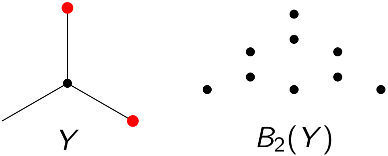

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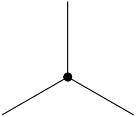

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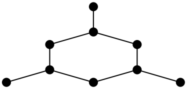

$B_2(Y)$

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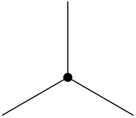

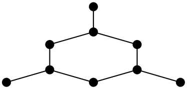

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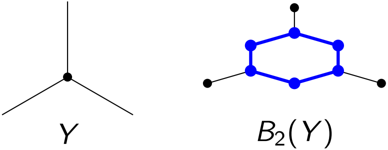

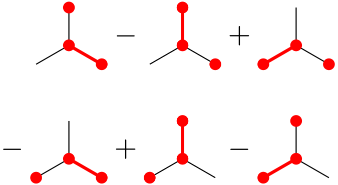

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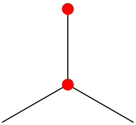

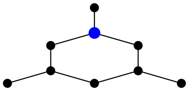

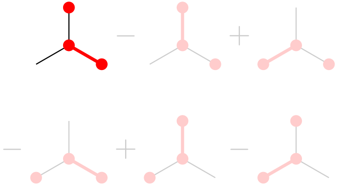

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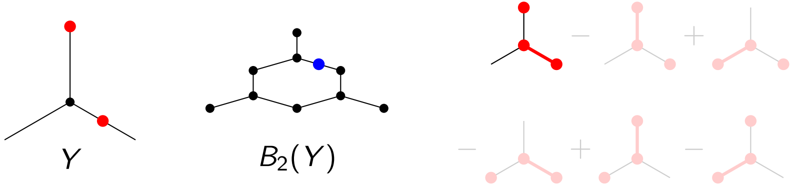

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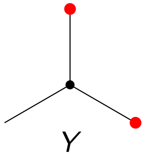

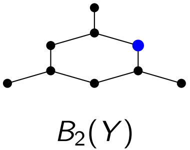

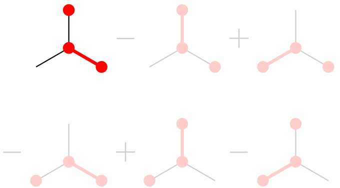

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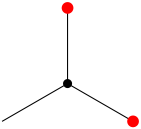

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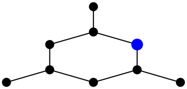

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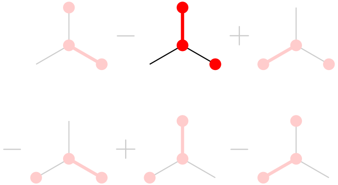

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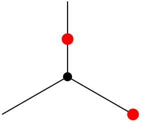

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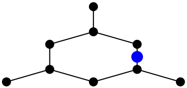

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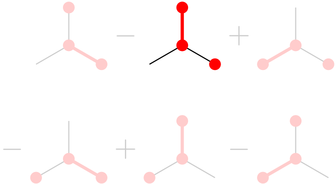

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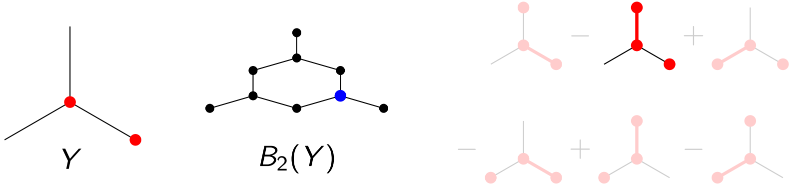

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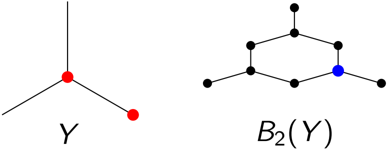

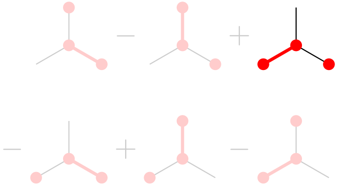

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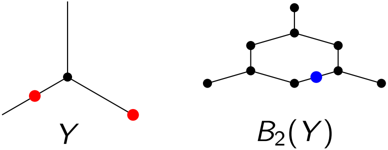

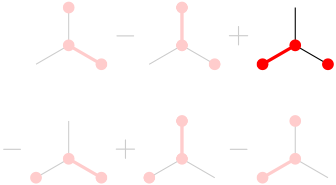

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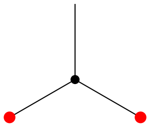

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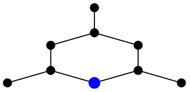

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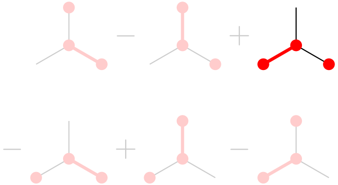

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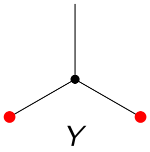

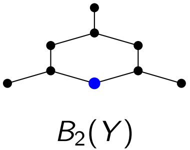

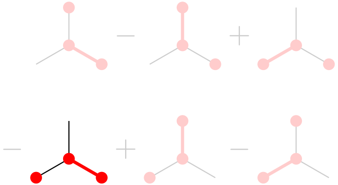

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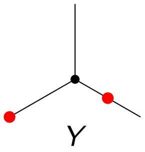

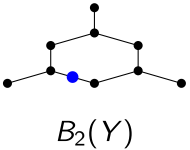

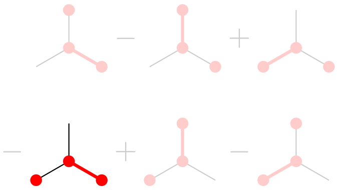

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Star classes

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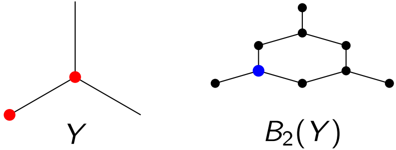

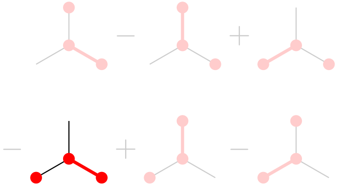

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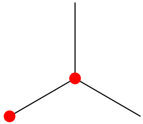

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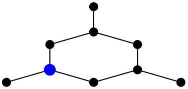

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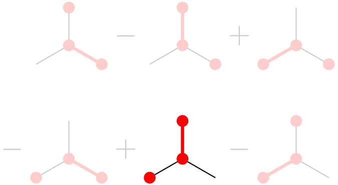

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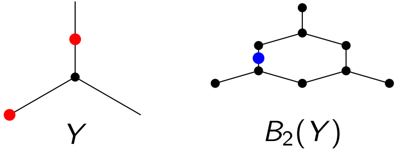

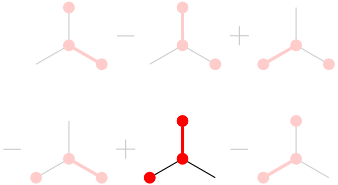

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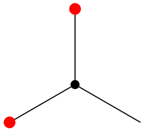

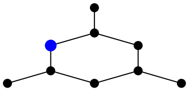

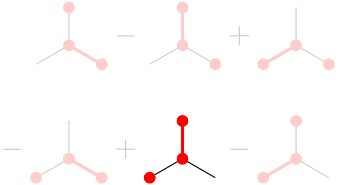

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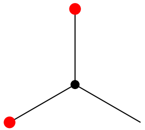

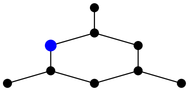

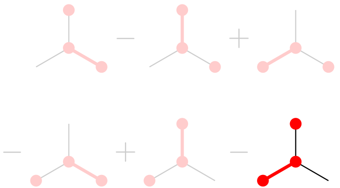

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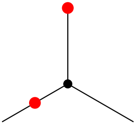

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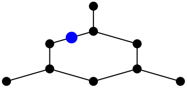

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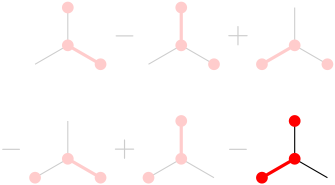

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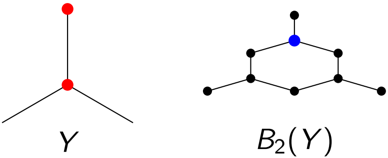

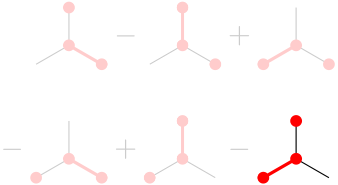

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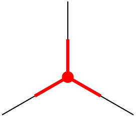

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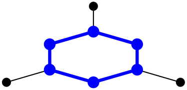

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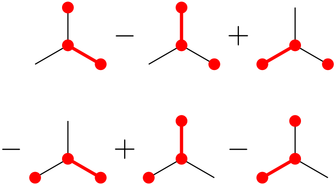

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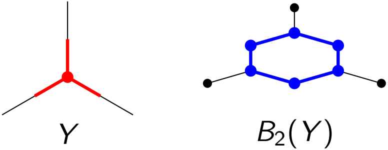

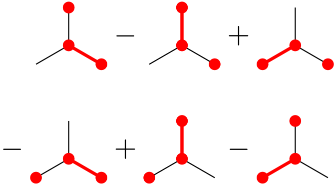

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Lemma

Star classes are nontrivial (they may be 2-torsion).

Ramos' connectivity invariant

Recall that over a field, $\dim H_i(B_k(\Gamma)))$ is eventually polynomial in k.

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Theorem (An–D.–Knudsen)

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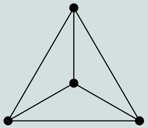
Examples of Ramos' invariant

Example



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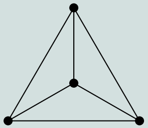

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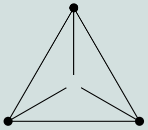

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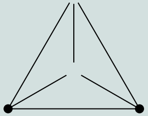

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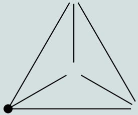

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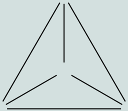

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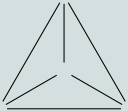

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≥ 5	$-\infty$	$-\infty$	0	all k

Lower bound: $N_\Gamma^i \geq \Delta_\Gamma^i - 1$

Idea to prove lower bound

Find a torus α of star classes at $W \subset V$ whose stabilizations grow quickly.

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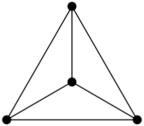

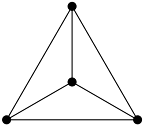

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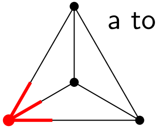

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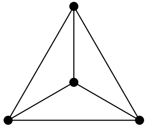

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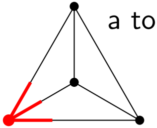

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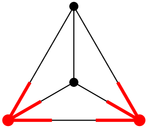

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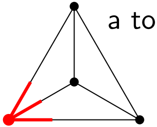

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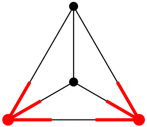

Lemma

If each star touches two or more components of $\Gamma \setminus W$, then α is rigid.

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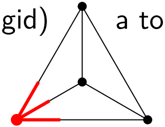

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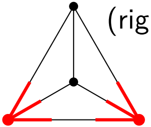

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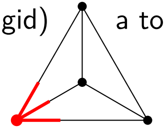

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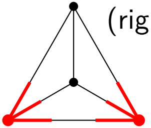

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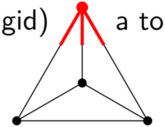
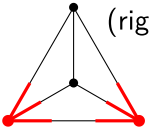
(not rigid)  a torus α with $|W| = 1$

a torus α with $|W| = 2$  (rigid)

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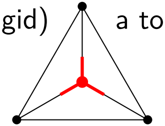

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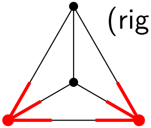

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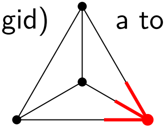
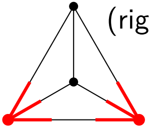
(not rigid)  a torus α with $|W| = 1$

a torus α with $|W| = 2$  (rigid)

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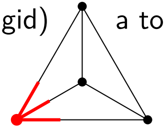

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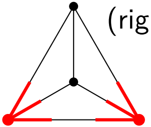


(rigid)

Lemma

If each star touches two or more components of $\Gamma \setminus W$, then α is rigid.

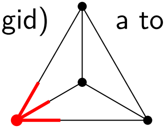
I.e., every representative of α contains generators blocking all of W.

Lower bound: $N_\Gamma^i \geq \Delta_\Gamma^i - 1$

Idea to prove lower bound

Find a torus α of star classes at $W \subset V$ whose stabilizations grow quickly.

(not rigid)



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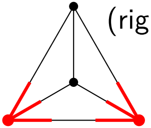

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For $|W| \neq 1$, if W realizes this maximum then every vertex of W touches two or more components of $\Gamma \setminus W$.

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Then we can build a rigid torus α. □

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Use rigidity at vertices to decompose $H_i(B(\Gamma))$.

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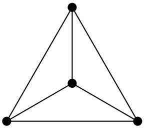

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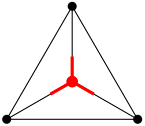

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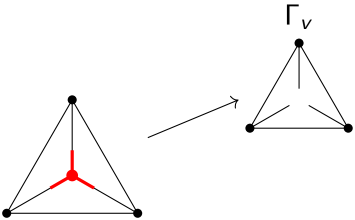

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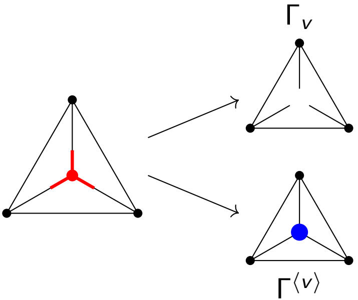

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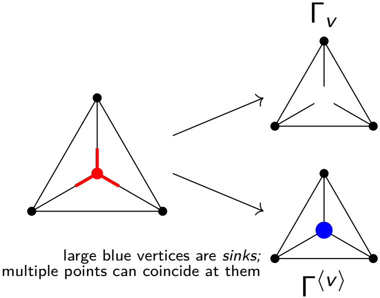

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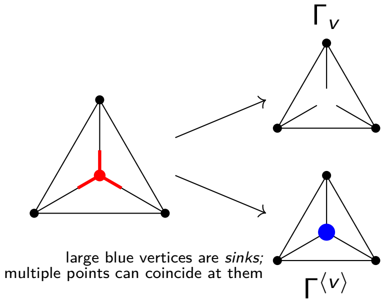

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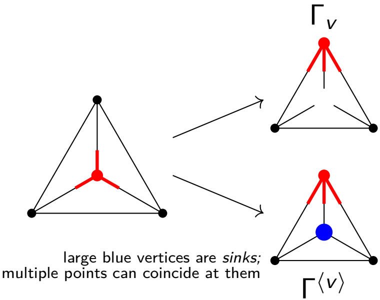

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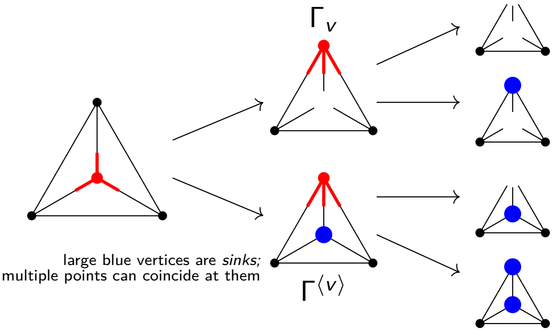

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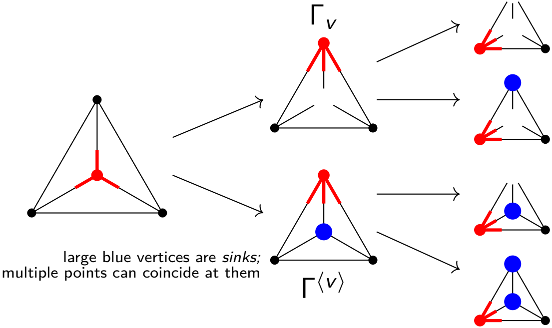

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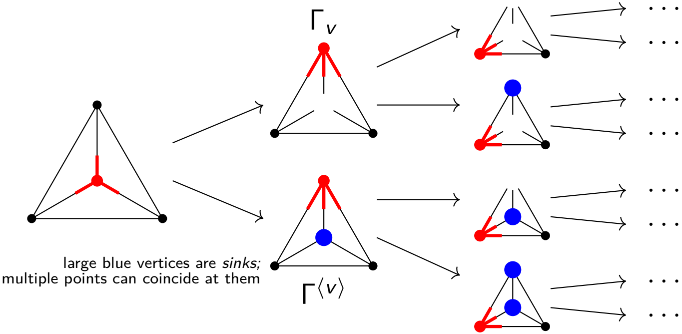

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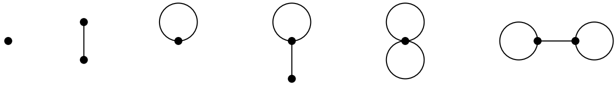
But with the lies I told, you can do well enough to get an upper bound.

Formality

Theorem

$C_*(B_*(\Gamma))$ is $\mathbb{Z}[E]$-formal if and only if Γ is small.



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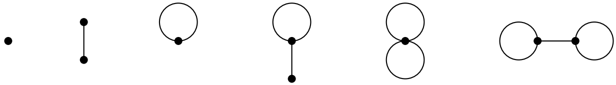

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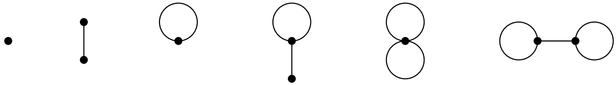

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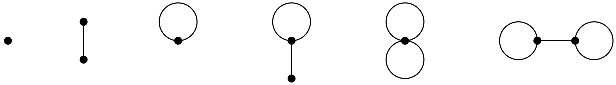

Question

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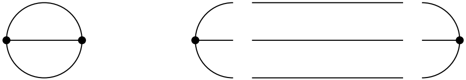
- Computation: assembling graphs
- Higher invariants

(Non)-formality and graph assembly



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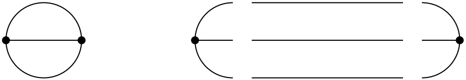

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If we had formality, we could conclude:

$$H_*B(\Gamma_1 \sqcup_{E'} \Gamma_2) \cong \mathrm{Tor}_*^{\mathbb{Z}[E']}(H_*B(\Gamma_1), H_*B(\Gamma_2)).$$

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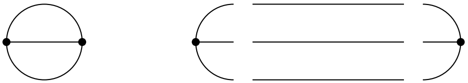

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Instead this is just page two of a Künneth spectral sequence in general.

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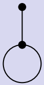

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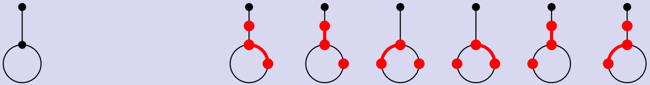

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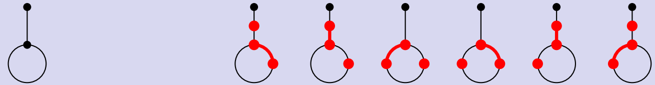

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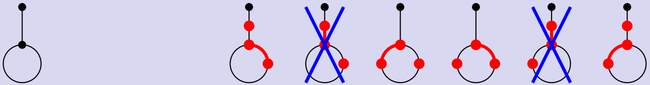

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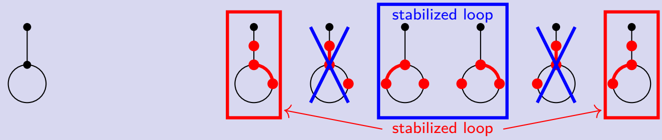

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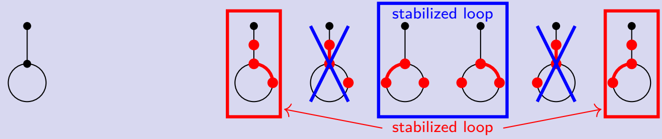

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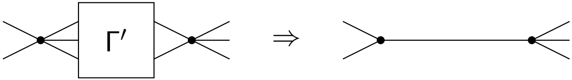

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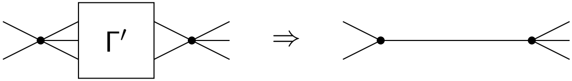

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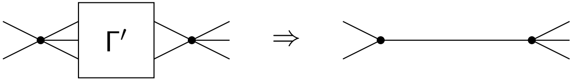

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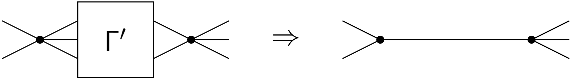

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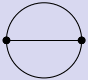

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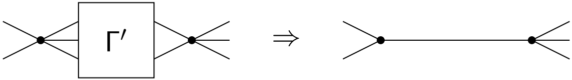

Lemma

Surgery reflects non-formality.

Lemma

Any large graph can be surgered to one of the following.

Building rigid indecomposable star classes

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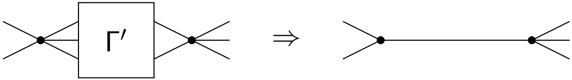

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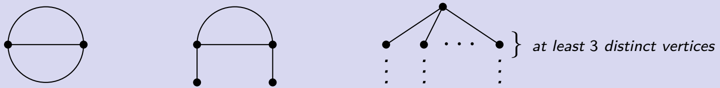

Using surgery

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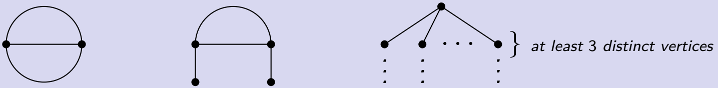

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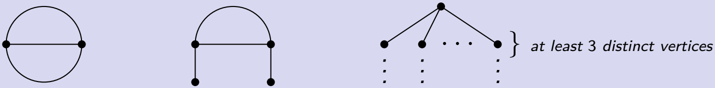

Lemma

In all three cases we can build a rigid indecomposable star class.

Using surgery

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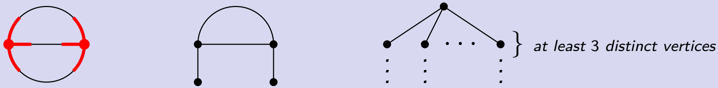

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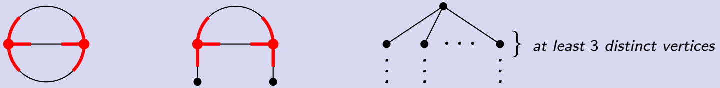

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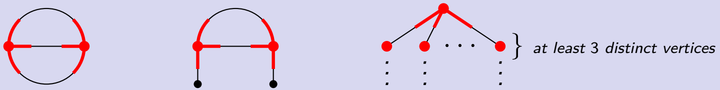

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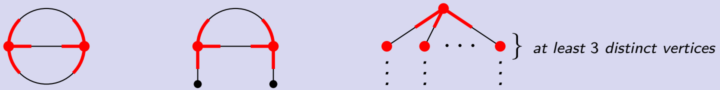

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Corollary

Large graphs are non-formal.

Thank you for your attention.