Oberwolfach

Gabriel C. Drummond-Cole

June 11, 2009

1 T. Schick

[If it's not pouring, we'll have the group photo after the second talk.] This is the first of two talks. It turns out that this is maybe the second in a series of three because of Dan Freed. I'd like to recall what a smooth cohomology theory (a smooth refinement of cohomology) is. This is a repetition of yesterday. The idea is to combine cohomology and differential forms which via the de Rham theorem represent real homology classes. The main diagram is that you have the homology $H^*(M)$ and the differential forms, the closed ones $\Omega^*(M)_{d=0}$, and you can pass to de Rham cohomology $H^*_{dR}(M)$, and the question is what you put in the corner to combine the information.

$$\begin{tikzcd} ? & H^*(M) \arrow[d] \\ \Omega^*(M)_{d=0} \arrow[r] & H^*_{dR}(M) \end{tikzcd}$$

and what you fill with is the smooth cohomology $\hat{H}(M)$, which should have a map to ordinary homology as well as a curvature map to closed forms:

$$\begin{tikzcd} \hat{H}^*(M) \arrow[r, "I"] \arrow[d, "R"'] & H^*(M) \arrow[d] \\ \Omega^*(M)_{d=0} \arrow[r] & H^*_{dR}(M) \end{tikzcd}$$

There should also be more. We will want an exact sequence, $\hat{H}(M) \xrightarrow{I} H^*(M)$ should be surjective, and what is the kernel? We should have an action a, action of forms, which gives a smooth cohomology class from a form of one degree less, and this will be modulo the image of d, so we have

$$H^{*-1}(M) \longrightarrow \Omega^{*-1}(M)/im\ d \xrightarrow{a} \hat{H}^*(M) \xrightarrow{I} H^*(M) \longrightarrow 0$$

I want to say more, each smooth cohomology class determines a smooth differential form. We can ask what happens from the image of a? Well, it's just the de Rham differential

$$\begin{tikzcd} H^{*-1}(M) \arrow[r] & \Omega^{*-1}(M)/im\ d \arrow[r, "a"] \arrow[dr, "d"'] & \hat{H}^*(M) \arrow[r, "I"] \arrow[d, "R"] & H^*(M) \arrow[r] & 0 \\ & & \Omega^*(M)_{d=0} & & \end{tikzcd}$$

I want to transform this observation into a definition.

Definition 1 *Given a cohomology theory H^*, a smooth refinement $\hat{H}^*$ is a functor from manifolds to graded Abelian groups with these transformations I, R, and a, with the given properties. You've already seen in Dan's talk that it's better to do this with generalized cohomology theories. Then maybe you have to think of a couple of things, in particular the comparison from integral to de Rham cohomology. To make this correct, you need to turn H to E. Well, forms have to be replaced by forms with values in V, where V are the coefficients made real, that is, $E^*(pt) \otimes \mathbb{R}$, this is the real cohomology. So we have instead*

$$\begin{tikzcd} E^{*-1}(M) \arrow[r] & \Omega^{*-1}(M,V)/im\ d \arrow[r, "a"] \arrow[dr, "d"'] & \hat{E}^*(M) \arrow[r, "I"] \arrow[d, "R"] & E^*(M) \arrow[r] & 0 \\ & & \Omega^*(M,V)_{d=0} & & \end{tikzcd}$$

That's an important bit of structure that you have, a ring structure. So let me add this:

Definition 2 *If E^* is multiplicative, we say $\hat{E}$ is multiplicative if it's not just a functor of Abelian groups but also graded rings and the transformations are compatible with the ring structure. Here this means that a is a module morphism, that $a(\omega) \cup x$, that should be the same as $a(\omega \wedge R(x))$ for forms and smooth cohomology classes ω and x.*

We will come to that a little later, you could ask given another cohomology theory, can you construct smooth refinements, or multiplicative refinements. There is another bit of structure to introduce. Once you have a multiplicative map, you want a wrongway (pushdown) map, but as a basic input, I want a very special such map for the product with S^1.

Definition 3 *$\hat{E}$ has S^1 integration if there is a natural transformation $\int_{S^1} : \hat{E}^*(M \times S^1) \to \hat{E}^{*-1}(M)$ compatible with integration of forms and for E. For E, if it's multiplicative, S^1 has a trivial tangent bundle, you have an orientation, but for E you have the suspension isomorphism. So it works without a multiplicative structure, even. There should also be, well, the classical property is that if you pull back from M and then push down you get zero, so that $\int \circ p^* = 0$ for $p : M \times S^1 \to M$ the projection on the first factor. There is a second property that we always have, in S^1 you have complex conjugation, so $\int id \times (z \mapsto \bar{z})^* = -\int$ because it is orientation reversing.*

The examples we have mentioned that deal with ordinary cohomology satisfy all of these things.

[Is it easy to say what the integral is in the ordinary cohomology?]

With the Cheeger-Simons model it's not so bad, with the Deligne model it's not so easy, so then it's a reason you like to have different models, but then you'd like to know that they are the same somehow.

Let me just play with these axioms a tiny little bit. Let me state a lemma.

Lemma 1 *Given a smooth $\hat{E}$, a smooth cohomology theory, you should immediately be suspicious that this is not a cohomology theory because of the differential form part. Is there a replacement for it? So there is a homotopy formula, if $h : M \times [0,1] \to N$ is a smooth homotopy, then $h_1^*(x) - h_0^*(x)$ is well, you have $h^*(R(x))$, which is now a differential form, well, so you can integrate this over $M \times [0,1]/M$ and then apply a. I need to know that I can use manifolds with boundary.*

Corollary 1 *Out of these functors you can produce a new one, and if you play with this on the kernel of R, this term on the right vanishes, so the kernel of R is homotopy invariant.*

Because of this observation, it deserves a name. So we call this the "flat" part of the smooth cohomology (because it is the kernel of the curvature.

How do we get the homotopy formula? To show the force of the axioms, as usual let's just show that the two inclusions of M into $M \times [0,1]$ satisfy this. It suffices to show that if you pull back:

$$i_1^*(x) - i_0^*(x) = a \int_{M \times [0,1]/M} R(x)$$

If x is pulled back by projection, if $x = p^*y$ then the left hand side is zero by functoriality, and the right hand side is a form on $M \times [0,1]$ is a form which is the pushdown of a pullback and so we get 0 without even taking a.

If x is general, there exists, the projection map p is a homotopy equivalence so at the level of usual homology you get a homomorphism, and you can lift the thing that pulls back by surjectivity, so you can get some y with $x - p^*(y)$ maps by I to 0, so $x - p^*y = a(\omega)$. Now you can play with Stokes' theorem.

You see from the axioms you've used the exact sequence in two places, surjectivity and knowledge of the kernel. What does Stokes tell us? If we restrict ω to two different sides of our cylinder, that's the same as $\int_[0,1]d\omega$. On the other hand, $d\omega$ is $Ra(\omega)$, and for $a\omega$ we can substitute and get that this is $\int R(x - p^*y)$, and by linearity this is $\int R(x)$, which is what shows up on the right hand side of our formula. Now we can finish: we want to compute $i_1^*(x) - i_0^*(x)$, and we might as well add the pullback of y here, so this can replace x with $x - p^*y = \omega$ so this is $i_1^*(a\omega) - i_0^*(a\omega) = a(\int R(x))$.

One proof. It's a good thing, hopefully.

Okay, let's come to the question, there are other things [unintelligible]that satisfy these axioms.

Oh, I forgot, let's do one calculation. Let's try to calculate ordinary cohomology $\hat{H}^1$ of a point. Look at this sequence, you know the forms on a point, so it turns out to be $\mathbb{R}/\mathbb{Z}$, and the same is true for $\hat{K}^1(pt)$. This is just an indication that in our models, they're built to be the home of invariants. These smooth versions are homes of refined invariants, and they're homes for secondary invariants. Maybe you can get to a flat theory of a point which is the home of secondary invariants. We don't have much but you have a reinterpretation and framework. If you just work in a homotopy theoretic context, well, you can't do much, but if you have different models you can figure things out.

Theorem 1 *Hopkins-Singer*
Precursors by Freed-Hopkins, saying that for each E^* as on the middle boards an $\hat{E}^*$ exists. Moreover, the flat part of the theory, you've seen it, it's important, and in this construction, $\hat{E}^*_{flat} = E^{*-1}(\,, \mathbb{R}/\mathbb{Z})$

Remark 1 *It's not at all evident how to obtain more structure like multiplication. It's even difficult that these are Abelian groups. I don't say that you can't construct these but you would have to work hard.*

It's at least convenient to have models, but this other structure, it's not easy, it's not made explicit.

Now let me add more information about that and a theorem.

Theorem 2 *This is done in an abstract way using quite a bit of homotopy theory. Using geometric models, multiplicative smooth extensions with S^1 integration are constructed for*

- *K-theory (Bunke, Schick), based on local index theory, so the cycles you can think of for $\hat{K}(M)$ are family index problems characterized by M, Dirac operators with the right kind of equivalence. We'll learn about that a little bit later.*
- *unitary bordism (Bunke, Schröder, Schick, Wiethhaup) and from there Landweber exact cohomology theories*

We have many models, how do they compare? It's good to have models to do different kinds of constructions. You want them to be compatible to be the same. We then come to the uniqueness question. How many smooth extensions are there?

Theorem 3 *Assume that E^* satisfies $E^2 n + 1(pt) \otimes \mathbb{Q} = 0$ and some technical assumptions (countable generation). Then any two smooth extensions $\hat{E}^*$ and $\tilde{E}^*$ with S^1 integration are* *naturally isomorphic, and there is a unique isomorphism compatible with integration. If the extensions of E have ring structure, then this is automatically an isomorphism of ring valued functors.*

Just to put this into context, if we don't require compatibility with the S^1 integration, then you could ask, is there more to say. Then the additive group structure might not be determined, there are exotic additive Abelian group structures on $\hat{K}^1$.

2 Kevin Costello

[The afternoon is devoted to two talks by Kevin, continuing his talks on factorization algebras.] I should start by mentioning something about yesterday's lecture. It's fine with me if you work with parameterized balls. I think this is an essentially equivalent set of axioms. I'll be working in the C^∞ complex so let's not worry about the holomorphic. So you can replace balls $B(M)$ with embeddings of $\bar{D}^n$ into M, and then the space of configurations is just a product, $Emb(\bar{D}^n, M) \times Emb(\bar{D}^n \sqcup \cdots \sqcup \bar{D}^n, \bar{D}^n)$.

[Isn't this more like framed little disks?] They both are, um, maybe, I don't know, you can pull it back. I want to present a theorem that will allow you to construct examples. The theorem is kind of nontrivial so the axioms are kind of unimportant.

I want to explain how to get this, what is the classical analog and how to I get one. I'm forseeing vociferous objections. The basic idea is the following. Factorization algebras and these form a symmetric monoidal category. This means that one can look for algebras over any operad in the category of factorization algebras. If F and F' are factorization algebras, then $(F \otimes F')(B) = F(B) \otimes F'(B)$. One can check that this is compatible with all structure, so the definition is that a classical factorization algebra is a commutative algebra in the category of factorization algebras. I like to think of a factorization algebra as being analagous to an E_n algebra but more general. Recall that an E_∞ object in E_n algebras is an E_∞ algebra. The extra commutativity gets rid of all the noncommutativity in E_n. The idea, I want to associate a classical factorization algebra to a classical field theory.

So suppose you have a classical field theory, meaning that the space of fields is sections on a bundle $E \to M$, so $S : \Gamma(M, E) \to \mathbb{R}$ is the classical action and it's local, so it's the integral of a Lagrangian. So the fundamental object of classical field theory is the space of solutions to the Euler Lagrange equations. So if $B \subset M$ is a ball, let $EL(B)$ be the set of sections, fields on the interior of B, which satisfy the Euler Lagrange equations. This is where I was expecting objections, because this is a subtle differential equation. Once you make this small, the analysis goes away.

The classical factorization algebra $\mathscr{F}_S$ associated to the classical action S assigns to B, the algebra $\mathscr{O}(EL(B))$ on the set of solutions to the Euler Lagrange equation. In the examples these will be formal power series, so don't worry about convergence. We want to make this into a factorization algebra, so we want maps $\mathscr{F}_S(B_1) \otimes \cdots \otimes \mathscr{F}_S(B_n) \to \mathscr{F}_S(B_{n+1})$ if B_1 through B_n form a disjoint subset of B_{n+1}. A solution of the Euler Lagrange equation on B

restricts to one on all of its subsets. So there's a map $EL(B_{n+1}) \to EL(B_1) \times \cdots \times EL(B_n)$. Taking functions on both sides reverses the arrows, so this yields the map $\mathscr{O}(EL(B_1)) \otimes \cdots \otimes \mathscr{O}(EL(B_n)) \to \mathscr{O}(EL(B_{n+1}))$.

If you look at functions on these, they should be compactly supported in the middle.

Let me give you an easy example. If the fields are C^∞ and $S(\phi)/\int_M \phi\Delta\phi$, then the Euler Lagrange equation is $\Delta\phi = 0$, so this is the space of harmonic functions on the ball. I want to emphasize that these are on the interior of the ball.

The action is quadratic so the space of solutions is linear. This is a closed subspace of the Frechet space of all smooth functions. So you take the product

$$\mathscr{O}(EL(B)) := \prod_{n\geq 0} Hom(EL(B)^{\otimes n}, \mathbb{R})^{S_n}$$

where Hom means continuous linear maps and $\otimes$ is the completed tensor product. Later we'll see that what we really want to do is take the derived space of solutions to the Euler Lagrange equations. But for now these formal power series will do.

When we do classical mechanics we have a Poisson algebra that we want to quantize, and here we'll find something like this too. Factorization algebras are a symmetric monoidal algebra, so I want to say that a factorization algebra is an algebra in the category of factorization algebras. There is an operad like this. There is such an operad, the E_0 operad has $E_0(0)$ a point and $E_0(n) = \emptyset$ otherwise. So an E_0 algebra in vector spoces is a vector space with an element. There's an axiom for factorization algebras, that they need to have a unit. I forgot to mention this, a section of F on $B(M)$, which is a unit for the product. The map for the unit object is specified by the unit element. So an E_0 algebra in factorization algebras is just a factorization algebra. What we'll be interested in is finding an algebraic structure in the classical case that will quantize to an E_0 structure.

We know that Poisson algebras, well, here is a chart:

Classical	Quantum
	E_0
Poisson	E_1, that is, associative
Commutative with a bracket of degree 1	E_2
Com with a bracket of degree 2	E_3

So we should fill this in with, for E_0, a commutative algebra and a bracket of degree +1. In general there's a filtration and the associated graded of the right side is the left side. For E_2 the homology is the left column, and so on going down. For a unifying picture, the guys on the left hand side are some degeneration of the right hand side.

This is in cochain complexes.

So Beilinson and Drinfeld define an operad over the ring $\mathbb{R}[[\hbar]]$ as being generated by a commutative product, a Poisson bracket of degree +1, with differential $d(\bullet) = \hbar\{\,,\,\}$. Let's call BD operad. It's clear that modding out by $\hbar$ this is the operad of commutative algebras with a Poisson bracket of degree $+1$, but if we invert $\hbar$, then the homology is trivial. I have two generators and the differential takes one to the other. This is an operad over the formal disk that interpolates between the Poisson guy and zero.

Aside: people say that BV is a framed E_2 operad. Unfortunately, E_2 has nothing to do with the Batalin Vilkovisky formalism. There was annoyance in the early nineties. This is the operad of the Batalin Vilkovisky formalism.

This is a long digression. Why should we try to quantize in this way? I need to introduce terminology for this operad. A P_0 operad is the operad of unital commutative Poisson algebras with a bracket of degree $+1$. So $P_0 = BD/\hbar$. Any time you take the critical scheme of anything you get one of these. A general fact is, suppose you have a manifold and F is a function on it, $f : M \to \mathbb{R}$, then functions on the derived critical locus of F is a P_0 algebra.

What is the derived critical locus? The critical locus is the zeros of df, $Z(df)$, so $\mathscr{O}$ of the critical locus is $\mathscr{O}(M)/Image(\Gamma(M, TM) \overset{\vee df}{\to} C^\infty(M))$. The derived critical locus has functions the dga

$$\to \Gamma(M, \wedge^2 TM) \to \Gamma(M, TM) \to \mathscr{O}(M)$$

This is the same as polyvector fields, where $\wedge^k TM$ is degree $-k$ and the differential is $\vee df$. The polyvector fields have the Schouten bracket, which is of degree plus one. This wants to quantize to an E_0 algebra.

I have two minutes left. We need to quantize this. The procedure of quantizing is the same as replacing F with a solution to the quantum master equation. If M has a measure then the derived critical locus, functions on it, $\mathscr{O}(Crit^n(f))$ has a canonical quantization to an E_0 algebra, an algebra over BD The quantization means that you need to give an order two operator whose deviation from being a derivation of the product is the bracket. So this is $(\Gamma(M, \wedge TM), \vee df + \hbar\Delta)$. This Δ arises when M has a measure.

So $\Delta X = Div\ X$, and a similar formula extends it.

3 Kevin Costello Part III

[They remind us that they can't do this bijection between napkins and folders if you don't put the napkins back.]

So far, we had the derived critical locus which had this extra structure of a P_0 algebra, suggesting that it wants to quantize to E_0. If we have a classical field theory, we expect that the derived space of solutions to the Euler Lagrange equation will yield the same structure but in the category of factorization algebras. This is the kind of thing that wants to become just a factorization algebra. This story is supposed to be the quantum field theory analog of the familiar deformation quantization story.

So let's just to make sure we're on the same page, do some examples of the derived space of solutions to the Euler Lagrange equations. If φ is a function and $S\varphi = \int \varphi\Delta\varphi$ then the derived solution space is the two term complex

$$\underbrace{C^\infty(M)}_{0} \xrightarrow{\Delta} \underbrace{C^\infty(M)}_{1}$$

If B is a ball in M then $\mathscr{O}(EL^n(B))$ is

$$\prod_{n\geq 0} Hom((C^\infty(Int\ B) \xrightarrow{\Delta} C^\infty(Int\ B))^{\otimes n}, \mathbb{R})^{S_n}$$

This is a commutative dga and defines a commutative factorization algebra. If $S(\varphi) = \int \varphi\Delta\varphi + \varphi^3$, we get the same algebra of functions, but the differential changes. The final example will be Yang Mills theory, which is more subtle. Before we take our derived critical scheme, we need to quotient by gauge symmetries, a quotient of $\Omega^1(M) \otimes \mathfrak{g}$ by $\Omega^0(M) \otimes \mathfrak{g}$ and then we take the dervied critical locus of the Yang Mills action. Let me explain what you get when you do this. This is known as the BV formalism. You take the derived space of solutions to the Yang Mills equation. So what we get is, well, when linearized, it just looks like the stuff we quotient by in degree -1 (M is four dimensional) and it's

$$\begin{array}{ccccccc} \Omega^0(M) & \xrightarrow{d} & \Omega^1(M) & \xrightarrow{d*d} & \Omega^3(M) & \xrightarrow{d} & \Omega^4(M) \\ \\ -1 & & 0 & & 1 & & 2 \end{array}$$

all tensored with $\mathfrak{g}$.

At some stage I'll mention a theorem that you can [unintelligible], that relies on the fact that there aren't many Poisson deformations.

I suppose the first thing to mention is

Theorem 4 *If we take the derived space of solutions to the Euler Lagrange equation, looking infinitessimally near a given solution, then functions on this has a Poisson structure in factorization algebras on M*

There are some technical restrictions, ellipticity of the quadratic term.

We'd like to quantize one of these, in the classical setting, the role the action plays, the action gives the differential. So this amounts to quantizing the action S into what's called a solution to the quantum master equation.

I've written a book about how to do this. The quantum master equation is not defined. You need an effective field theory setup. The book on my website, you kind of need, like, the machinery of counter terms, Wilsonian effective action, and so on, to even talk about the quantum master equation. I don't want to get into this, just impress people.

Theorem 5 *Joint with O. Guilliam*
"Naive" version: consider the scalar field theory with an action

$$S(\varphi) = \int \varphi(\Delta\varphi + m^2\varphi)$$

along with arbitrary local cubic and higher terms. Let $\mathscr{F}_S$ be the classical factorization algebra associated to it. This is a P_0 algebra.

Let $Q^{(n)}(\mathscr{F}_S)$ be the set of quantizations defined mod $\hbar^{n+1}$. Then this is lifts of $\mathscr{F}_S$ to an algebra over $BD/\hbar^{n+1}$.

There is a sequence $T^{(n)} \to T^{(n-1)} \to \cdots \to T^{(1)} \to pt$ where $T^{(n)}$ maps to $Q^{(n)}(\mathscr{F}_S)$. There is some space of quantizations. I want to tell you how big this space is. Each $T^{(n)}$ is a torsor over $T^{(n-1)}$ for the Abelian group of local functionals of φ. There's no canonical way to quantize, but at each level we're free to add any Lagrangian.

So if we look at $T^{(\infty)}$, this is isomorphic to series in $\hbar$, $\sum_{k\geq 1} \hbar^k S^{(k)}$ where $S^{(k)}$ is a local functional. But this is noncanonical.

A more sophisticated version would be to say, well, let's consider families parameterized by simplices. Consider any reasonable classical theory, yielding a classical factorization algebra $\mathscr{F}$. Let $Q^{(n)}(\mathscr{F})$ be the simplicial set of possible quantizations. Consider families parametrized by cochains on a simplex. I want to say that this is a torsor modelled on the cochain complex. What is this in terms of Lagrangians and actions? $Der_{loc}(\mathscr{F})$ is the cochain complex of locally defined derivations of $\mathscr{F}$. To think in more concrete terms, this preserves P_0 structure, that is, it is, in fact local functionals on an "extended" space of fields.

The second theorem says roughly that there exists a sequence of simplicial sets $T^{(n)} \to T^{(n-1)} \to \cdots \to T^{(1)} \to pt$ with maps $T^{(n)} \to Q^{(n)}(\mathscr{F}_S)$ such that each $T^{(n)}$ fits into a homotopy fiber diagram

$$\begin{tikzcd} T^{(n)} \arrow[r] \arrow[d] & 0 \arrow[d] \\ T^{(n-1)} \arrow[r, "Obs"] & Der_{loc}(\mathscr{F})[2] \end{tikzcd}$$

Theorem 6 *Let $\mathfrak{g}$ be a simple Lie algebra. Then there is an essentially unique quantization of Yang Mills theory on $\mathbb{R}^4$ which is essentially unique once we impose the condition that it is renormalizable in the Wilsonian sense, it behaves well with respect to scaling, and the set of such quantizations is $\hbar\mathbb{R}[[\hbar]]$.*

If you're a physicist, you might say the essence of quantum field theory is correlation functions. If $\mathscr{F}$ is a factorization algebra in the quantum sense corresponding to some quantum field theory, then we should think that $\mathscr{F}(B)$ is the set of observables, observations we can make on B. The correlation functions, well, if B_1 and B_2 are disjoint inside of B, then the structure of factorization algebra has a very natural interpretation, namely, $\mathscr{F}(B_1) \otimes \mathscr{F}(B_2) \to \mathscr{F}(B)$, that's doing both observations. We would like to have correlation functions, so these should be cochain maps from, well, a bunch of disjoint balls, I can get joint expectation values, so maps from these $\mathscr{F}(B_1) \otimes \cdots \otimes \mathscr{F}(B_n) \to \mathbb{R}$ for the balls disjoint. If $O_i \in \mathscr{F}(B_i)$ then the correlation function should measure the probability that all the observations happen, how they correlate.

We'd also like some compatibility conditions. If you have two balls, then you can put your two observables into the bigger ball, and you should get the same thing whether you put them in one ball or two. So the diagram should commute, which is

$$\begin{tikzcd}
\mathscr{F}(B_1) \otimes \cdots \mathscr{F}(B_n) \arrow[r] \arrow[d] & \mathbb{R} \\
\mathscr{F}(\tilde{B}) \otimes \mathscr{F}(B_3) \otimes \cdots \mathscr{F}(B_n) \arrow[ur] &
\end{tikzcd}$$

So far, this is just a hope. But here, the correlation functions are often uniquely determined by this property. We can consider correlation functions with coefficients in any cochain complex, and require that they satisfy the equation.

Definition 4 *Beilinson-Drinfeld, Kevin Walker, Jacob Lurie The chiral homology $CH(M, \mathscr{F})$ equals the homotopy universal recipient of correlation functions.*

So this is the colimit over all diagrams like this. The space of correlation functions, in many good cases, is one dimensional.

Lemma 2 *For a massive scalar field, the chiral homology is isomorphic te $\mathbb{R}[[\hbar]]$. You might say that's not very interesting. We're interested in gauge theories. But you wouldn't expect that this would give you a number at the end. In general, $CH(M)$ looks like measures on the space of critical points of the classical action.*

So if we start with an isolated critical point, you get something one dimensional.

In this situation, correlation functions exist and are unique. In general the correlations will define a measure in the space of classical solutions