

# Dennis Sullivan Course Notes

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You have two copies of the plane, one including the origin and the other infinity, and connect them by inversion.

... a vector field wraps around twice so the number is two here.

A  $2n - 1$  sphere over  $M^{2n}$  the obstruction will be in  $H^{2n}(M^{2n}, \pi_{2n-1}S^{2n-1})$ , which is  $\mathbb{Z}$ . This is the Euler characteristic of  $M$ .

There's a great picture in Steenrod's book. You have a triangulation of the manifold, and you get one top dimensional cell for each triangle in the triangulation. You make a vector field which goes down a dimension. At one point in the middle of each face there's a singularity, and you move away from the center toward the edge and corner.

In general, at a  $k$ -simplex, you have  $k$  inputs and  $n - k$  outputs. So if you remove these points, you'll have a cross-section. Then you'll have an index, an integer for extending it over.

Anyway, up to the overall sign, this obstruction is the sum over all the simplices of  $-1$  to the dimension of the simplex. The exercise is, compute the degree. Take a  $k$ -dimensional subspace and  $n - k$  and make those directions unit in and out. Extend linearly. Then map the unit sphere to the unit sphere and check for degree, which will be plus or minus one.

Class is over.